

Local Cone Structures on Varieties

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Introduction. In [1] it was shown that a real algebraic variety is locally a cone over a subvariety at an isolated singularity, and in [3], this result was generalized to give a cone theorem around any point. The object of this paper is to show that this local cone structure is unique in the sense that if a neighborhood of a point on a variety is expressed in two ways as a cone over a subvariety, then there is a deformation which carries the generators and base of the one cone into those of the other.

1. Statement of main theorem. Let \mathfrak{V} be a bunch of real algebraic varieties in an open set of some Euclidean space satisfying the following conditions:
(1) If V_1 and V_2 are in \mathfrak{V} , so are all the components of $V_1 \cap V_2$.
(2) If V_1 is in \mathfrak{V} and V_2 is the smallest real algebraic variety containing all singularities of V_1 , then all the components of V_2 are in \mathfrak{V} .

Condition (2) could also be stated by saying that the components of the singular subvariety of V_1 are in \mathfrak{V} , where it is understood that the singular subvariety is determined by the Jacobian conditions applied to generators of the ideal of V_1 . It has to be remembered that all the points of this subvariety are not necessarily singular in the local analytic sense.

Associated with the bunch \mathfrak{V} , a stratification can be defined as follows. Write Σ_k for the set of all varieties V_i in \mathfrak{V} of dimension $\leq k$, and define

$$S_k = \Sigma_k - \Sigma_{k-1}.$$

Then S_k is to be the k -dimensional stratum of the required stratification. Note that the connected components of S_k are open subsets of varieties in \mathfrak{V} and are at the same time real analytic manifolds.

The next step is to define an admissible curve family relative to the bunch \mathfrak{V} . Essentially the idea is to define a family having the property of the conclusion of Theorem 4.1 in [3].

Definition 1.1. Let \mathfrak{V} be a bunch as above in the open set U of a Euclidean space. A family w of analytic curves is to be called admissible relative to \mathfrak{V} if the following conditions hold. Let $|\mathfrak{V}|$ be the point set union of components of \mathfrak{V} .

(1) There is exactly one member of w through each point of $U \cap |\mathfrak{V}|$.

(2) If a member of w has a point in common with a variety of \mathfrak{V} , it lies entirely in that variety. In view of the condition satisfied by \mathfrak{V} , this means that each member of w lies in some stratum of the stratification associated with \mathfrak{V} .