

# On $p$ -Sidon Sets

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**Introduction.** Let  $G$  be a compact abelian group with dual group  $\Gamma$ . A subset  $E$  of  $\Gamma$  is  $p$ -Sidon ( $1 \leq p < 2$ ) if there is a constant  $\alpha$  such that each  $\phi$  in  $C(G)$  with  $\hat{\phi}$  supported on  $E$  satisfies  $\|\hat{\phi}\|_p \leq \alpha \|\phi\|_\infty$ . Hence a set is 1-Sidon if and only if it is Sidon. Moreover a duality argument yields that  $E$  is  $p$ -Sidon if and only if  $\ell^{p'}(E) \subset M(G)^\wedge|_E$ , where the latter symbol denotes the restrictions of the Fourier-Stieltjes transforms to  $E$  and where  $p' = p/(p - 1)$ . Several of the basic results on  $p$ -Sidon sets were independently obtained by Bożejko and Pytlik [1], L.-S. Hahn [4], and Edwards and Ross [3]. The article of Edwards and Ross appears to contain all that was known about  $p$ -Sidon sets prior to this paper. Here we prove:

**Theorem.** *Suppose  $A_1, \dots, A_n$  are mutually disjoint, infinite subsets of  $\Gamma$  whose union is dissociate. Then  $E \equiv A_1 + \dots + A_n$  is  $p$ -Sidon if and only if  $p \geq 2n/(n + 1)$ .*

For  $n = 2$  this theorem is due to Edwards and Ross [3, Corollary 5.5] and is one of the main contributions of their paper.

Let  $\alpha_p$  denote the class of  $p$ -Sidon sets, for  $1 \leq p < 2$ . Suppose  $1 \leq p_1 < p_2 < 2$ . Clearly  $\alpha_{p_1} \subset \alpha_{p_2}$ . The question as to whether or not the containment is proper is certainly one of the fundamental questions of the theory. The result of Edwards and Ross implies that if  $1 \leq p_1 < 4/3 \leq p_2 < 2$ , then  $\alpha_{p_1}$  is properly contained in  $\alpha_{p_2}$ .

Perhaps most importantly it shows that there exists  $p$  ( $1 < p < 2$ ) such that  $\alpha_p$  is distinct from the class  $\alpha_1$  of ordinary Sidon sets. Our extension to arbitrary  $n \geq 2$  of the Edwards-Ross result implies that an infinite number of the collections  $\alpha_p$  are distinct. The general question of whether  $\alpha_{p_1}$  is always properly contained in  $\alpha_{p_2}$  for  $1 \leq p_1 < p_2 < 2$  remains unresolved.

Our proof of the theorem above is divided into two sections. In the first section we begin by proving (Lemma 1) that the sum of any  $n$  infinite sets is never  $p$ -Sidon for  $p < 2n/(n + 1)$ . The lemma is a fairly direct extension of Corollary 2.7 of [3], but its proof does offer some additional technical difficulties and so we include it. One direction of our theorem follows immediately from