Enumeration of Liftings in the Non-Principal Case

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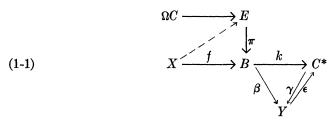
1. Introduction. If X and Y are spaces, let [X; Y] be the set of homotopy classes of maps $X \to Y$.

Let $\pi: E \to B$ be a fibration with fiber ΩC , where C is an H-space, and let $f: X \to B$ be a map, where X is a CW-complex. The problem we consider in this paper is enumeration of the inverse image of [f], the class containing f, under $\pi_*: [X; E] \to [X; B]$.

The case where π is a principal fibration has been discussed by James and Thomas [3], Nomura [7], and [4] (the only case considered in the latter is where C is an Eilenberg-MacLane space.). In [3], oriented real n-bundles over an n-complex, and real n-bundles over an n-complex for odd n, are classified up to oriented-bundle, and bundle equivalence, respectively. In this paper (Thm. 4.1) we classify real n-bundles over an n-complex for even n.

Remark 1.2 below is a special case of Theorem 1 of [7], and is simply a restatement of Theorem 1.2 of [3], without the hypothesis that B be an H-space; also of 3.2 of [4] without the hypothesis that C be an Eilenberg-MacLane space. For instructional purposes, the proof is repeated here.

In the current paper, a large class of cases is considered in which π is not a principal fibration. The problem shall be represented by the following diagram:



where β and γ are fibrations and $\gamma \cdot k = \beta$, ϵ is a section of γ , $\gamma^{-1} * = C$ (we take * to be the base-point of any space), $\gamma^{-1}y$ is a *H*-space with identity ϵy for all $y \in Y$, where the multiplication is continuous on $(\gamma \times \gamma)^{-1}\Delta Y \subset C^* \times C^*$, Y is path-