

# Enumeration of Liftings in the Non-Principal Case

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**1. Introduction.** If  $X$  and  $Y$  are spaces, let  $[X; Y]$  be the set of homotopy classes of maps  $X \rightarrow Y$ .

Let  $\pi: E \rightarrow B$  be a fibration with fiber  $\Omega C$ , where  $C$  is an  $H$ -space, and let  $f: X \rightarrow B$  be a map, where  $X$  is a  $CW$ -complex. The problem we consider in this paper is enumeration of the inverse image of  $[f]$ , the class containing  $f$ , under  $\pi_*: [X; E] \rightarrow [X; B]$ .

The case where  $\pi$  is a principal fibration has been discussed by James and Thomas [3], Nomura [7], and [4] (the only case considered in the latter is where  $C$  is an Eilenberg–MacLane space.). In [3], oriented real  $n$ -bundles over an  $n$ -complex, and real  $n$ -bundles over an  $n$ -complex for odd  $n$ , are classified up to oriented-bundle, and bundle equivalence, respectively. In this paper (Thm. 4.1) we classify real  $n$ -bundles over an  $n$ -complex for even  $n$ .

Remark 1.2 below is a special case of Theorem 1 of [7], and is simply a restatement of Theorem 1.2 of [3], without the hypothesis that  $B$  be an  $H$ -space; also of 3.2 of [4] without the hypothesis that  $C$  be an Eilenberg–MacLane space. For instructional purposes, the proof is repeated here.

In the current paper, a large class of cases is considered in which  $\pi$  is not a principal fibration. The problem shall be represented by the following diagram:

$$(1-1) \quad \begin{array}{ccccc} \Omega C & \longrightarrow & E & & \\ & \nearrow \text{dashed} & \downarrow \pi & & \\ X & \xrightarrow{f} & B & \xrightarrow{k} & C^* \\ & & \searrow \beta & \nearrow \gamma & \nearrow \epsilon \\ & & & & Y \end{array}$$

where  $\beta$  and  $\gamma$  are fibrations and  $\gamma \cdot k = \beta$ ,  $\epsilon$  is a section of  $\gamma$ ,  $\gamma^{-1} * = C$  (we take  $*$  to be the base-point of any space),  $\gamma^{-1}y$  is a  $H$ -space with identity  $\epsilon y$  for all  $y \in Y$ , where the multiplication is continuous on  $(\gamma \times \gamma)^{-1} \Delta Y \subset C^* \times C^*$ ,  $Y$  is path-