

Multi-Parameter Spectral Theory

PATRICK J. BROWNE

Communicated by RALPH PHILLIPS

1. Introduction. Let H_1, H_2, \dots, H_k be separable Hilbert spaces and $H = \bigotimes_{1 \leq r \leq k} H_r$ their tensor product. In each space H_r we assume we have continuous operators $T_r, V_{rs} : H_r \rightarrow H_r, s = 1, 2, \dots, k$ such that

- (i) all the operators $T_r, V_{rs}, r, s = 1, 2, \dots, k$ are Hermitian,
- (ii) for any choice of $f_r \neq 0, f_r \in H_r, r = 1, 2, \dots, k,$

$$(1) \quad \det (V_{rs} f_r, f_r)_r > 0$$

where $(\cdot, \cdot)_r$ denotes the inner product in H_r .

Such a system has, in effect, been the basis for discussion of multi-parameter eigenvalue problems by Atkinson [2] and Browne [4]. The results obtained have been applied to multi-parameter systems of second order ordinary differential equations thus covering the work of Fairman [6]. In these investigations the operators V_{rs} have been assumed compact relative to the operators T_r or alternatively, the operators T_r have been self adjoint with compact resolvent. The differential equation application has been extended to cover so called "singular" cases—see Browne [5]. Here the operators T_r are of Sturm-Liouville type over a half line and thus while being self adjoint do not necessarily have compact resolvent. This suggests the possibility of a more general multi-parameter spectral theory in which the compactness assumptions are removed. It is the aim of this paper to develop, at least partially, such a theory.

2. Operators from determinantal tensor expansions. Following Atkinson [2] we define the operators $\Delta_s : H \rightarrow H, s = 0, 1, \dots, k$ as follows. Let $f = f_1 \otimes \dots \otimes f_k$ be a decomposable element of H and let $\alpha_0, \alpha_1, \dots, \alpha_k$ be arbitrary complex numbers. Then $\Delta_0 f, \dots, \Delta_k f$ are defined by the equation

$$\sum_{s=0}^k \alpha_s \Delta_s f = \bigotimes \begin{vmatrix} \alpha_0 & \alpha_1 & \dots & \alpha_k \\ T_1 f_1 & V_{11} f_1 & \dots & V_{1k} f_1 \\ \dots & \dots & \dots & \dots \\ T_k f_k & V_{k1} f_k & \dots & V_{kk} f_k \end{vmatrix}$$