

On Nonstationary Stokes Flow Past an Obstacle

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1. Introduction. Solutions of the exterior stationary problem for the Stokes equations do not generally tend to the prescribed limit at infinity sufficiently fast for the difference between a solution and its limit to be square-summable. The square-integral of this difference, which can be interpreted as the energy present in a flow, is known to be infinite in the case of Stokes flow past an obstacle to which fluid adheres. Therefore, however natural it may be in some respects to base the study of nonstationary solutions upon energy estimates, such studies are restricted to a class of nonstationary solutions (finite energy solutions) which tend more rapidly than do stationary solutions to their limits at infinity. In this paper we study a class of nonstationary solutions of the Stokes equations whose behavior at infinity is merely required to be similar to that of stationary solutions which possess a finite Dirichlet integral. Our purpose is to provide a natural solution class for the initial boundary value problem in the case of an exterior domain, to treat existence and uniqueness questions within this class, and finally to prove the convergence of nonstationary solutions to stationary ones as time $t \rightarrow \infty$. The last mentioned objective amounts to demonstrating the attainability of stationary solutions as limits of nonstationary solutions and it has been our primary goal. In earlier papers [3, 4] we studied similar problems for the nonlinear Navier–Stokes equations and in those papers our results were limited in scope by finite energy hypotheses which are dispensed with here for the Stokes equations.

Since energy estimates are impossible for the solutions of problems we will consider, we base our methods instead upon L^2 estimates for certain combinations of a solution's derivatives. The solutions we obtain belong to a Hilbert space H_0 whose norm involves these derivatives and which is defined by completion of a set of stationary solutions. A close connection thus results between the stationary and nonstationary theories. Section 2 below contains a preliminary investigation of the space H_0 .