

Transformations of Flows by Discrete Random Measures

DONALD GEMAN & JOSEPH HOROWITZ

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0. Introduction. Let $\mathfrak{D} = (\Omega, \mathfrak{F}, P, \theta_t)$ be a dynamical system and $\alpha(dt, \omega)$ a homogeneous random measure on the real line \mathbf{R} , i.e. $\alpha(B, \omega)$ is measurable for each $B \in \mathfrak{B}$, is finite on compacts and homogeneous relative to \mathfrak{D} : for almost every $\omega \in \Omega$,

$$(1) \quad \alpha(B + t, \omega) = \alpha(B, \theta_t \omega), \quad t \in \mathbf{R}, \quad B \in \mathfrak{B},$$

with \mathfrak{B} the Borel σ -field on \mathbf{R} . A random time $\tau : \Omega \rightarrow \mathbf{R}$ will be called a “translation time” if the transformation $\theta_\tau : \Omega \rightarrow \Omega$ given by $\theta_\tau(\omega) = \theta_{\tau(\omega)}(\omega)$ preserves \hat{P}_α -measure, \hat{P}_α being the Palm measure of α (all terminology will be explained momentarily). We deal in this paper with the question of when translation times exist.

Solutions to this problem are known in two special cases. The first is when α is a *counter*, i.e. for each ω , α places unit mass at time points $R_n(\omega)$ having no finite accumulation point, with $\dots < R_{-1} < R_0 \leq 0 < R_1 < \dots$; then each R_n , $n \in \mathbf{Z}$, is a translation time and $\hat{\theta}_n = \theta_{R_n}$ is a (discrete-time) flow on $\Omega_0 = \{\omega : R_0(\omega) = 0\}$ relative to \hat{P}_α , [13], [14], [15]. This set-up is called a *stationary point process* and is the classical one of the subject. The case of a marked point process is included here since, if L is a subset of the space of marks, the customary Palm measure \hat{P}_L is defined as the Palm measure of the counter ν_L which places unit mass at each time that a mark from L appears (see [6], [10]). The second case is when α is a *continuous* homogeneous measure, meaning that, a.s. α has no point masses. If, for $t \geq 0$,

$$\hat{\alpha}(t, \omega) = \inf \{s > 0 : \alpha((0, s], \omega) > t\}$$

(a similar definition is given for $t < 0$), then each $\hat{\alpha}(t)$ is a translation on time, and $\hat{\theta}_t = \theta_{\hat{\alpha}(t)}$ defines a flow $\Omega_0 = \{\omega : \hat{\alpha}(0, \omega) = 0\}$ relative to \hat{P}_α [9], [16]. In both cases there is an inversion formula retrieving P from \hat{P}_α , and, if $\alpha(\mathbf{R}_+) = \infty$ a.s., the “dual” dynamical system $\hat{\mathfrak{D}} = (\Omega_0, \mathfrak{F}_0, \hat{P}_\alpha, \hat{\theta})$, where $\mathfrak{F}_0 = \mathfrak{F} \cap \Omega_0$, shares with \mathfrak{D} the property of ergodicity. (These and related questions are discussed in [4].)