

On Some Paradoxes Concerning Two-Dimensional Stokes Flow Past an Obstacle

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1. Introduction. In the preceding paper [2] some problems for both the stationary and nonstationary Stokes equations are proved to be uniquely solvable within a certain class of generalized solutions. The results are the same for any number of spacial dimensions $n \geq 2$. In this paper we examine some paradoxes concerning these solutions which occur when $n = 2$; one is the Stokes paradox and the others are related to it but concern nonstationary problems. These paradoxes are all obtained as consequences of a single lemma which asserts that when $n = 2$ the function space H_0 that defines the class of solutions being considered contains elements which equal nonzero constants in a neighborhood of infinity. One may infer from the lemma that the boundary condition at infinity is not strictly imposed on solutions of this class when $n = 2$. Roughly speaking the paradoxes concern particular violations of the condition at infinity.

The Stokes paradox of hydrodynamics states that it is impossible to solve the Stokes equations in a two-dimensional exterior domain for a time independent solution which vanishes on the boundary and which tends to a nonzero limit at infinity. In Section 3 we consider a class of generalized solutions of the two-dimensional exterior stationary problem which includes the class H_0 solutions of [2]. Theorem 1 is a version of the Stokes paradox; it states that the unique generalized solution of this class vanishes identically. We also consider in Section 3 the initial boundary value problem for a two-dimensional exterior domain with an initial velocity which tends to a nonzero limit at infinity. Theorem 2 states that the class H_0 solution of this nonstationary problem converges to zero in every finite region as $t \rightarrow \infty$.

In Section 4 the initial boundary value problem for a two-dimensional exterior domain is studied with nonzero prescribed forces. Theorem 3 states that there exist square-summable forces such that the unique class H_0 solution of this problem violates the prescribed condition at infinity. This is particularly surprising because there also exists a unique finite energy solution of the problem which satisfies the condition at infinity.