

On the Cohomology of Fixed-Point Sets and Coincidence-Point Sets

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§1. This note contains generalizations of the fixed-point and coincidence-point theorems of Lefschetz, Brouwer, and Borsuk–Ulam. Here we state the main theorems and corollaries: all proofs are deferred till sections 2, 3, and 4. The symbol \check{H} denotes Čech cohomology, and \dot{H} denotes Alexander–Spanier cohomology. Manifolds are assumed connected throughout.

Theorem 1. *Let $f, g: M^m \times N_1^n \rightarrow N_2^n$ be continuous maps, where M^m, N_1^n and N_2^n are closed orientable manifolds. Choosing a point $t_0 \in M$, define $f_0 = f|_{t_0 \times N_1}, g_0 = g|_{t_0 \times N_1}$. Let $A = \{(t, x) \in M \times N_1 \mid f(t, x) = g(t, x)\}$, let $i_A: A \rightarrow M \times N_1$ denote inclusion and let $p_1: M \times N_1 \rightarrow M$ denote projection onto the first factor. If the Lefschetz coincidence index $L(f_0, g_0)$ is nonzero, then*

$$i_A^* \circ p_1^*: \check{H}^m(M; \mathbf{Z}) \rightarrow \check{H}^m(A; \mathbf{Z})$$

is nonzero. (Thus A has cohomological dimension $\geq m$.)

Corollary A. *Let M^m be a closed orientable manifold and N^n a compact orientable manifold, with possibly $\partial N^n \neq \emptyset$. If $g: M^m \times N^n \rightarrow N^n$ is such that the Lefschetz number $L(g_0) \neq 0$, and $A = \{(x, t) \in M^m \times N^n \mid g(x, t) = t\}$, then $i_A^* \circ p_1^*: \check{H}^m(M^m; \mathbf{Z}) \rightarrow \check{H}^m(A; \mathbf{Z})$ is nonzero.*

Corollary B. *If N^n is a compact manifold, and $f: [0, 1] \times N^n \rightarrow N^n$ is continuous map such that $f_0 = f|_{0 \times N}: N \rightarrow N$ has nonzero Lefschetz number, then the set $A = \{(t, y) \in [0, 1] \times N \mid f(t, y) = y\}$ has a component intersecting both $0 \times N$ and $1 \times N$.*

Corollary B was proved originally by F. Browder [1].

Some terminology is required to state the next theorem. For each $n \in \mathbf{Z}^+$, there is a (non-free) \mathbf{Z}_2 -action on \mathbf{R}^n generated by $\alpha: \mathbf{R}^n \rightarrow \mathbf{R}^n$, where $\alpha(x) = -x$. For n odd, $n \geq 3$ and $p \geq 3$, p prime, there are free “standard” (see §3) \mathbf{Z}_p -actions on \mathbf{S}^{n-2} which can be extended raywise to give \mathbf{Z}_p -actions on \mathbf{R}^{n-1} having 0 as the only fixed point.