

Restriction of H^p Functions to the Diagonal of \mathbf{U}^n

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Communicated by the Editors

Let \mathbf{C} denote the complex numbers, \mathbf{U} the open unit disc, \mathbf{T} the unit circle, and \mathbf{U}^n and \mathbf{T}^n the Cartesian product of n copies of \mathbf{U} and \mathbf{T} , respectively. Let dm_n and dA be the Lebesgue measures on \mathbf{T}^n and \mathbf{U} , respectively, normalized so that $dm_n(\mathbf{T}^n) = dA(\mathbf{U}) = 1$. For $0 < p < \infty$, define $H^p(\mathbf{U}^n)$ as the space of all analytic functions F on \mathbf{U}^n that satisfy

$$\|F\|_p^p = \sup_{0 < r < 1} \int_{\mathbf{T}^n} |F(r\omega)|^p dm_n(\omega) < \infty,$$

and define $H^\infty(\mathbf{U}^n)$ as the space of all bounded analytic functions on \mathbf{U}^n .

If $F \in H^p(\mathbf{U}^n)$, one may study F by considering its "slice functions". For each $\omega \in \mathbf{T}^n$, we define the slice function $F_\omega : \mathbf{U} \rightarrow \mathbf{C}$ by $F_\omega(\lambda) = F(\omega\lambda)$. The function F_ω is always analytic, but simple examples show that if $p < \infty$, then F_ω need not belong to $H^p(\mathbf{U})$. However, Rudin (see [6], pages 53 and 69) has proved that if $F \in H^p(\mathbf{U}^2)$ for $p = 1$ or $p = 2$, then the slice function f defined by $f(z) = F(z, z)$, or the "diagonalization" of F , always belongs to the Bergman space A^p of analytic functions in $L^p(dA)$. Rudin has then noted that the diagonalization map is actually onto A^2 and has asked whether this map is onto A^1 and also whether these results generalize to other spaces $H^p(\mathbf{U}^n)$. We now answer these questions in the affirmative.

Theorem. For $n \geq 1$, let D_n be the diagonal map, taking analytic functions on \mathbf{U}^n into analytic functions on \mathbf{U} by

$$D_n F(z) = F(z, z, \dots, z).$$

Then for $n > 1$, and for $1 \leq p < \infty$, D_n effects a bounded linear mapping of $H^p(\mathbf{U}^n)$ onto $A^{p, n-2}$, where $A^{p, n-2}$ is the space of analytic functions f on \mathbf{U} that satisfy

$$\|f\|_{p, n-2}^p = \int_{\mathbf{U}} |f(z)|^p (1 - |z|^2)^{n-2} dA(z) < \infty.$$