

Certain Topologies on the Space of Temperate Distributions and its Multipliers

JAN KUČERA & KELLY McKENNON

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Let \mathcal{S}' be the space of all temperate distributions on Euclidean space \mathbf{R}^m . An inductive topology $T_i(\mathcal{S}')$ on \mathcal{S}' was defined and exploited in [3]. This topology will be shown herein to be just the strong topology $\beta(\mathcal{S}', \mathcal{S})$, where \mathcal{S} is the space of rapidly decreasing functions on \mathbf{R}^m .

In [3] the topology $T_i(\mathcal{S}')$ led naturally to a construction of a locally convex space \mathcal{O} of multipliers. It evolved that \mathcal{O} , algebraically, was just \mathcal{O}_M . We shall show in this paper that \mathcal{O} and \mathcal{O}_M are the same topologically as well.

Notation. The set of non-negative integers and its m -fold product will bear the notation N and N^m respectively. For each $\alpha \in N^m$, $|\alpha|$ will mean $\sum_{i=1}^m \alpha_i$ and, for each $x \in \mathbf{R}^m$, x^α will mean $\prod_{i=1}^m x_i^{\alpha_i}$. For $\alpha \in N^m$, D^α will denote the differential operator $\partial^{|\alpha|} / \partial x_1^{\alpha_1} \cdots \partial x_m^{\alpha_m}$. The symbol C^∞ will represent the set of infinitely differentiable functions defined on \mathbf{R}^m . For each $\alpha \in N^m$, $\beta \in N^m$, and $f \in C^\infty$, we set

$$(1) \quad \|f\|_{\alpha, \beta} = \sup \{ |D^\alpha(f)(x)| \cdot |x^\beta| : x \in \mathbf{R}^m \}, \quad \alpha, \beta \in N^m.$$

The set $\{f \in C^\infty : \|f\|_{\alpha, \beta} < \infty \text{ for all } \alpha, \beta \in N^m\}$ is the space \mathcal{S} of rapidly decreasing functions. The family of norms on it given by (1) generates a topology $T(\mathcal{S})$. By $\{\|\cdot\|_n\}_{n=1}^\infty$ will be meant a fixed, but arbitrary, family of norms on \mathcal{S} which generates $T(\mathcal{S})$ and is such that $\|\cdot\|_{n+1} \geq \|\cdot\|_n$ for each $n \in N$.

For each $n \in N$, H^n will denote the completion of \mathcal{S} with respect to $\|\cdot\|_n$. For all $p, q \in N$, $\mathcal{O}_{p, q}$ will be the set of all functions f such that $fh \in H^q$ for all $h \in \mathcal{S}$ and $\|f\|_{p, q} = \sup \{ \|fh\|_q : h \in H^p, \|h\|_p \leq 1 \} < \infty$; thus, $\mathcal{O}_{p, q}$ may be viewed as a linear space of linear operators from H^p into H^q and will be given the topology generated by the norm $\|\cdot\|_{p, q}$. For each $q \in N$, the linear space $\bigcup_{p=1}^\infty \mathcal{O}_{p, q}$ will be written as \mathcal{O}_q and considered to bear the associated inductive limit topology. By \mathcal{O} will be meant the linear space $\bigcap_{q=1}^\infty \mathcal{O}_q$ endowed with the associated projective limit topology $T(\mathcal{O})$.

The set \mathcal{O}_M is the family of all functions $f \in C^\infty$ such that, for each $\alpha \in N^m$, there