

# *Spaces of Functions with Asymptotic Conditions on $\mathbb{R}^n$ .*

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*Communicated by S. S. CHERN*

**0. Introduction.** In many physical problems over unbounded regions in space, the boundary conditions include the asymptotic behavior of the solutions as  $x$  approaches infinity. For example, in fluid mechanics one often assumes that the flow becomes steady at infinity. In general relativity, the physically correct assumption for a noncompact universe with finite mass is that the curvature tensor falls off as  $1/R$ .

In order to apply the techniques of global nonlinear analysis to this type of problem, one must specify spaces of functions satisfying the desired asymptotic conditions and study the behavior of the particular differential operators given by the physical problem on these spaces. In this paper we discuss a collection of spaces which should be appropriate for a large variety of physical applications.

Of primary importance is the behavior of the Laplace operator. Surjectivity of the Laplacian is essential to existence theory in fluid mechanics (3) and general relativity. For bounded regions and compact manifolds the usual choices of spaces are the Sobolev spaces  $L_k^p$  or the Holder spaces  $C^{k+\alpha}$ ,  $0 < \alpha < 1$ . The Holder spaces involve no asymptotic conditions and so the Sobolev spaces may seem to be a natural choice. However, it is easily shown that the Laplace operator is not a surjective map from  $L_k^p$  to  $L_{k-2}^p$ .

In Section 1 of this paper the spaces  $M_{s,\delta}^p$  are introduced. These spaces consist of functions which approach 0 at infinity a bit more strongly than the functions in  $L_s^p$ . It is also shown that for some choices of  $p$ ,  $s$ , and  $\delta$ ,  $M_{s,\delta}^p$  has a continuous ring structure.

In Section 2 it is shown that for some  $p$ ,  $s$ , and  $\delta$ , the Laplacian is an isomorphism between  $M_{s,\delta}^p$  and  $M_{s-2,\delta+2}^p$ . This fact rests on a straightforward application of a pair of inequalities found in a recent paper of Nirenberg and Walker (5).

In Section 3 it is shown that a simple application of the previous results yields a Hodge (or Helmholtz) decomposition for some of these  $M_{s,\delta}^p$  spaces. There are some analogs of this result known for the Sobolev spaces  $L_s^2$ . In (4) it is