

On Scattering by Time-Dependent Perturbations

E. J. P. GEORG SCHMIDT

Communicated by RALPH PHILLIPS

1. Introduction. Let H be a Hilbert space. For $t \in \mathbf{R}$ (the real line), and for $j \in \{0, 1\}$ let $A_j(t)$ be self-adjoint operators in H , satisfying conditions which ensure that the initial value problems

$$dx_j(t)/dt = iA_j(t)x_j(t), \quad x_j(s) \text{ prescribed,}$$

always have unique solutions. Associated with these equations are 2-parameter families $U_j(s, t)$ of evolution operators defined by the requirement that $U_j(s, t)x_j(s) = x_j(t)$ for each solution $x_j(\cdot)$ of the equation. These operators must clearly satisfy the identities

$$U_j(s, s) = I, \quad U_j(r, t)U_j(s, r) = U_j(s, t).$$

One defines the two *wave operators* $W_{+s}(A_0(\cdot), A_1(\cdot))$ and $W_{-s}(A_0(\cdot), A_1(\cdot))$ in terms of strong limits by

$$W_{\pm s}(A_0(\cdot), A_1(\cdot)) = s - \lim_{t \rightarrow \pm \infty} U_1(t, s)U_0(s, t),$$

when these limits exist. If $\text{Range } W_{+s} = \text{Range } W_{-s}$ (in which case the wave operators are said to be *complete*), one defines the scattering operator $S^s(A_0(\cdot), A_1(\cdot))$ by

$$S^s(A_0(\cdot), A_1(\cdot)) = W_{+s}(A_0(\cdot), A_1(\cdot))^*W_{-s}(A_0(\cdot), A_1(\cdot))$$

(where the adjoint of an operator T is denoted by T^*).

The case when $A_j(t)$ are independent of t , so that $T_j(s, t) = \exp [iA_j(t - s)]$, has been extensively studied (see *e.g.* Kato [9], and the references given there). The definitions we have given, and the elementary properties we now list without proof, involve simple extensions of the familiar theory.

Proposition 1. a) *The existence and completeness of $W_{\pm s}(A_0(\cdot), A_1(\cdot))$ for all s follows from the existence and completeness for any particular value of s .*