

Backward and Forward Random Evolutions

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1. Introduction and preliminaries. Pinsky [7] considered a Markov process on N lines, $\xi(t) = (X(t), V(t))$ where $V(t)$ is a finite Markov chain on $\{v_1, \dots, v_N\}$ with stationary transition probabilities and $X(t)$ is the position of a particle moving on a line with velocity $V(t)$ at time t . The backward equation for this process is

$$\frac{\partial}{\partial t} u(t, x, v_i) = v_i \frac{\partial}{\partial x} u(t, x, v_i) + \sum_{j=1}^N q_{ij} u(t, x, v_j)$$

where $Q = (q_{ij})$ is the infinitesimal matrix of V . Griego and Hersh [4 and 5] considered a probabilistic approach to the study of equations of the form

$$(1) \quad \frac{\partial u_i}{\partial t} = A_i u_i + \sum_{j=1}^N q_{ij} u_j$$

that introduced the idea of random evolutions.

Suppose $V(t, \omega)$ is a right-continuous stationary Markov chain on $\{1, \dots, N\}$ with infinitesimal matrix $Q = (q_{ij})$. Let $\{T_i(t), t \geq 0, i = 1, \dots, N\}$ be a family of strongly continuous semigroups of bounded linear operators on a fixed Banach space B . A_i is the infinitesimal generator of T_i . Let $\tau_j(\omega)$ be the time of the j^{th} jump of $V(t, \omega)$ and $N(t, \omega)$ be the number of jumps up to time t .

A random evolution $\{R(t, \omega), t \geq 0\}$ is defined by the product

$$R(t) = T_{V(0)}(\tau_1) T_{V(\tau_1)}(\tau_2 - \tau_1) \cdots T_{V(\tau_{N(t)})}(t - \tau_{N(t)}).$$

Let \tilde{B} be the N -fold product of B with itself; $\tilde{f} = (f_1, \dots, f_N)$ is a typical element of \tilde{B} . Griego and Hersh defined a new semigroup $\tilde{T}(t)$ on \tilde{B} specified componentwise by $(\tilde{T}(t)\tilde{f})_i = E_i[R(t)f_{V(t)}]$, where E_i denotes expectation under the condition $V(0) = i$. The Cauchy problem for the system

$$\frac{\partial u_i}{\partial t} = A_i u_i + \sum_{j=1}^N q_{ij} u_j, \quad u_i(0+) = f_i$$

is solved by $\tilde{u}(t) = \tilde{T}(t)\tilde{f}$.