

Hadamard Multipliers of Infinite Matrices

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1. Introduction. This paper discusses various cases of the following problem:

1.1 Given a class L of infinite matrices $F = (F_{ii})$, determine the class $M(L)$ of all matrices G such that $G \cdot F = (G_{ii}F_{ii})$ is in L whenever F is in L .

A particular case of 1.1 is the diagonal problem:

1.2 If $F \in L$, is the matrix $\Delta F = \text{diag}(F_{11}, F_{22}, \dots)$ in L ?

The latter problem has been treated in various papers [5], [9], [10] and [11]. Problem 1.1 is a special case of the general problem of multipliers of sequence spaces; see *e.g.* [4] and [6].

In this paper we consider $M(L)$ for L one of the following spaces:

- (a) $L = S \otimes_{\theta} T$, θ is a K -homogeneous crossnorm.
- (b) L consists of all continuous, integral, compact or nuclear matrix transformations from S into T .

In both cases we assume S and T are BK -spaces in which the coordinate functionals $\{e_i\}$ form an unconditional basis. Our main results are that in case (a) $M(L)$ contains all matrices of integral transformations from ℓ_1 into m (Theorem 2.1) and that in case (b) $M(L)$ contains all diagonal block matrices (including I) (Theorem 4.2).

We recall some language about topological tensor products. Let X and Y be Banach spaces, and $X \otimes Y$ their tensor product. Two important norms on $X \otimes Y$ are the *greatest crossnorm*

$$\gamma(u) = \inf \left\{ \sum_n \|x_n\| \|y_n\| : \sum_n x_n \otimes y_n = u \right\}$$

and the *least crossnorm*.

$$\lambda(u) = \sup \left\{ \left\| \sum_n f(x_n)y_n \right\| : f \in X', \|f\| \leq 1 \right\}.$$

The completion of $X \otimes Y$ under the norm $\gamma(\lambda)$ is denoted by $X \otimes_{\gamma} Y$ ($X \otimes_{\lambda} Y$) (p. 65 of [2]).