

Homomorphisms of $L^\infty(G)$ and Liftings

BENJAMIN B. WELLS, JR.

Communicated by the Editors

I. Introduction.

1.1. We will show a close connection between the subject of liftings of Haar measurable functions on a locally compact group and some recently defined functions arising from taking complex homomorphisms of the translates of essentially bounded functions.

Let G denote throughout an infinite locally compact group. The algebra of all bounded measurable complex functions defined on G , modulo those that vanish *a.e.* with respect to left Haar measure m , will be denoted by $L^\infty(G)$, or simply by L^∞ . The norm $\|f\|_\infty$ of a function f in L^∞ is the essential supremum of $|f|$. The algebra of all bounded measurable complex functions everywhere defined on G will be denoted by $M^\infty(G)$. The norm (still symbolized $\|f\|_\infty$) in $M^\infty(G)$ is the supremum of the values of $|f|$. As usual the space of all complex measurable functions f on G such that the integral of $|f|$ with respect to m is finite is denoted by $L^1(G)$.

A Haar measurable set A in a locally compact group G is said to be *locally null* if $m(K \cap A) = 0$ for every compact $K \subseteq A$. In the sequel the statement that a property holds for almost all $x \in G$ (*a.e.*) shall mean that the set where it fails is *locally null*. We shall write $f \equiv g$ if $f = g$ *a.e.* and $A \equiv B$ if the characteristic functions of A and B are equal *a.e.*

Properly speaking, a function f in L^∞ is the equivalence class of functions everywhere defined on G and equal to f *a.e.* John von Neumann showed that in the case of G equal to the unit interval it is possible to select exactly one function from each of the equivalence classes in such a way that the selection is algebraic, *i.e.* is linear and products of selected functions are precisely those functions selected from the product of the equivalence classes. D. Maharam [2] and subsequently A. I. Tulcea and C. I. Tulcea [7] generalized von Neumann's result to various contexts.

1.2. To be precise, a mapping $\rho: M^\infty(G) \rightarrow M^\infty(G)$ is called a *lifting* if the following axioms are satisfied.

$$(1) \quad \rho(f) \equiv f$$