

Minimal Embedding-Reflective Subcategories in the Category of Hausdorff Spaces

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All categories are assumed to be full subcategories of the category \mathfrak{Top} of all topological spaces. A subcategory \mathfrak{A} of a category \mathfrak{L} will be said to be embedding-reflective in \mathfrak{L} if it is epireflective in \mathfrak{L} (see [3, p. 76]) and each epireflection map is an embedding. It is not hard to show that the property of being an object of an embedding-reflective subcategory is a universal property in the sense of [7], and hence that the collection of spaces which possess a universal property is an embedding-reflective subcategory.

I. 1.1. In [7] Van der Slot characterized the embedding-reflective subcategories of \mathfrak{J} , the category of Tychonov spaces, as precisely those subcategories which are closed under products and the taking of closed subsets (*i.e.* epireflective in \mathfrak{J}) and which contain all compact Hausdorff spaces. Then the collection of all compact Hausdorff spaces forms an embedding-reflective subcategory of \mathfrak{J} which is minimal, both in the sense that it contains no proper subcategory which is embedding-reflective and in the sense that it is contained in every embedding-reflective subcategory. (We have not been able to determine whether or not these two notions of minimality are distinct.) Since epireflective subcategories of \mathfrak{JC} , the category of all Hausdorff spaces, are closed under the taking of closed subsets (see [3, Thm. 10.2.1]) it follows that any embedding-reflective subcategory of \mathfrak{JC} must contain all H -closed spaces, but in [4] it was shown that the only reflective subcategory of \mathfrak{JC} containing all H -closed spaces is \mathfrak{JC} itself. Hence \mathfrak{JC} contains no proper embedding-reflective subcategories. Many examples are known of categories between \mathfrak{JC} and \mathfrak{J} which possess proper embedding-reflective subcategories (for example, the category of all m -ultracompact spaces is embedding-reflective in the category of subspaces of m -ultracompact spaces [7]), however it is generally not known which categories contain minimal (in either sense) embedding-reflective subcategories. We now show the existence of a category of Hausdorff spaces containing \mathfrak{J} which possesses a minimal