

A One Phase Stefan Problem

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Introduction. A one phase Stefan problem is the description, typically, of the melting of a body of ice, maintained at zero degrees centigrade, in contact with a region of water. Given are an initial temperature distribution of the water, the heat of fusion, and the energy contributed to the system. The unknowns are the temperature distribution as a function of space and time, and the free boundary, which consists of the ice-water interface. The temperature function is required to solve the heat equation in the region of water for each time, and energy is conserved across the interface. More generally, one considers a two phase Stefan problem where the temperature distribution satisfies one heat equation in the region of water, another in the region of ice, and energy is conserved across the interface.

This paper is the study of a one phase problem. Recently G. Duvaut [7] has introduced a formulation of this problem as a parabolic variational inequality. A solution to the variational inequality is then interpreted as a generalized solution to the Stefan problem. We follow his approach here. After a few historical remarks, our conclusions are described below.

When the space dimension is $n = 1$, the two phase problem has been completely solved. A solution pair Θ, Γ , with Γ a continuous curve, was found in Friedman [11]. Very recently, Cannon, Henry, and Kotlow [6] have shown that Γ is continuously differentiable. The one phase problem in one dimension had been solved earlier by several authors; we refer to [8].

As for the problem in several space variables, Kamenomostkaja [14] and one of the authors [10] have proved the existence and uniqueness of a weak solution to the two phase problem. In [10] it was further shown that, in the case of one phase, the free boundary has no interior points.

Here, as we have mentioned, the problem is investigated in terms of a variational inequality. It involves a boundary condition depending on the time t which tempts one to consider it in the setting of an evolution inequality with time dependent cone $K(t)$. However, the results known in this setting (Brezis [4], Bensoussan–Friedman [1]) are not sufficient for our purposes. But it turns