

A Characterization of Complex Space Forms in Terms of Bochner Curvature Tensor and Chern Classes and its Application

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1. Introduction. In this paper, we consider complex manifolds of dimension ≥ 2 . A Kaehler manifold of constant holomorphic sectional curvature is called a *complex space form*. The tensor field introduced by S. Bochner [1] is considered as a complex version of the conformal curvature tensor and is called the *Bochner curvature tensor* (definition will be given in §2). It was proved by S. Tachibana [5] that an Einstein Kaehler manifold with vanishing Bochner curvature tensor is a complex space form.

Let M be a Kaehler manifold with fundamental 2-form Φ and Ricci 2-form γ_1 . We say that M is *cohomologically-Einstein* if $[\gamma_1] = a \cdot [\Phi]$ for some a , where $[\ast]$ denotes the cohomology class represented by \ast . It is well-known that the first Chern class $c_1(M)$ is represented by γ_1 . The class of cohomologically-Einstein Kaehler manifolds is wider than the class of Einstein Kaehler manifolds.

As a generalization of Tachibana's result, it is natural to consider cohomologically-Einstein Kaehler manifolds with *vanishing Bochner curvature tensor*. Under some assumption on the second Chern class $c_2(M)$, we obtain a characterization of compact complex space forms.

Theorem 1. *Let M be an n -dimensional compact Kaehler manifold. If*

- (i) $c_1(M) = a_1 \cdot [\Phi]$
- (i) $c_2(M) = a_2 \cdot [\Phi^2]$
- (iii) $2(n + 1)a_2 \geq na_1^2$
- (iv) *the Bochner curvature tensor is zero,*

then M is a space form.

If M is a compact complex space form, then (i), (ii), (iv) are satisfied and equality holds in (iii).

Let $P_{n+p}(\mathbf{C})$ be an $(n + p)$ -dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature 1. An n -dimen-