

# Analytic Embeddings in Logmodular Algebras

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**Introduction and summary of results.** Suppose  $A$  is a function algebra on the compact Hausdorff space  $X$ . Let  $M_A$  denote the maximal ideal space of  $A$ . If  $A$  is a logmodular algebra on  $X$ , then the non-point Gleason parts of  $M_A$  carry analytic structure. Let  $H^\infty(U)$  be the set of bounded analytic functions on  $U$ , the open unit disk in the complex plane. In the theorem below,  $P$  carries the relative weak-star topology of  $M_A$  which is inherited from  $A^*$ , the dual space of  $A$ .

**Theorem.** (Hoffman [4]) *Let  $X$  be a compact Hausdorff space and suppose  $A$  is logmodular on  $X$ . If  $P \subset M_A$  is a part containing more than one point, then there is a 1-1 continuous map  $\Psi$  of  $U$  onto  $P$  such that  $\bar{f} \circ \Psi(z) \in H^\infty(U)$  for  $f \in A$ .*

The map  $\Psi$  is unique in the sense that if two maps  $\Phi$  and  $\Psi$  satisfy the conclusions of Hoffman's theorem, then  $\Phi^{-1} \circ \Psi$  is a conformal self-map of the disk. Wermer [8] earlier had proved this theorem when  $A$  is a Dirichlet algebra on  $X$ . See Stout [7], p. 184 for further references.

In this paper we study the map  $\Psi$  with the added assumption that  $\Psi$  is a homeomorphism of  $U$  onto  $P$ , where  $P$  has the relative weak-star topology of  $M_A$ . When  $\Psi$  is a homeomorphism of  $U$  onto  $P$  that satisfies the conclusions of Hoffman's theorem, we will call  $\Psi$  an *analytic embedding*. If  $X$  is metric and  $\bar{P} \setminus P \subset X$  (where  $\bar{P}$  is the weak-star closure of  $P$  in  $M_A$ ), we show that  $\Psi$  extends radially to  $T$ , the unit circle, to define an essentially 1-1 map  $\psi$  of  $T$  into  $X$ . We show that  $\psi$  is measurable in the sense that  $\psi^{-1}(V)$  is Lebesgue measurable for Borel sets  $V$  and that  $\bar{f} \circ \psi(e^{it}) \in H^\infty(U)$  for  $f \in A$ . We use  $\psi$  to characterize the representing measures supported on  $X$  for points of  $P$  and to gain information about the annihilating measures of  $A$ .

**§1. Notations and definitions.** Let  $X$  be a compact Hausdorff space and denote by  $C(X)$  the set of complex-valued continuous functions on  $X$ . We say that  $A$  is a *function algebra* on  $X$  if  $A$  is a uniformly closed point-separating subalgebra of  $C(X)$  which contains the constant functions. Equipped with the