

# *Disappearing Solutions for the Dissipative Wave Equation*

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*Communicated by* RALPH PHILLIPS

**Introduction.** In this work we study a special class of solutions for the dissipative wave equation in either a bounded or exterior region  $\Omega$  with smooth boundary  $\partial\Omega$ . The systems we study have the form

$$\begin{aligned} \square u + c(x)u_t &= 0 \\ (1.1) \quad \frac{\partial u}{\partial n} + \gamma(x)u_t + \sigma u|_{\partial\Omega} &= 0, \quad \sigma \geq 0 \\ u(0) &= f_1 \\ u_t(0) &= f_2. \end{aligned}$$

The dissipation of energy in the system in (1.1) is a consequence of the requirements  $c(x) \geq 0$  and  $\gamma(x) \geq 0$ . We also assume that  $c$  and  $\gamma$  are smooth functions and that  $c$  vanishes near infinity. If one introduces the energy norm for the Cauchy data  $f = (f_1, f_2)$  by

$$\|f\|_E^2 = \int_{\Omega} |\nabla f_1|^2 + \int |f_2|^2 + \int_{\partial\Omega} \sigma |f_1|^2,$$

then it follows in a standard fashion that the mappings  $T(t)f \equiv (u(t), u_t(t))$  for  $t \geq 0$  form a contraction semigroup in the energy norm with infinitesimal generator  $A$  given by

$$A = \begin{bmatrix} 0 & 1 \\ \Delta & c \end{bmatrix}$$

on a domain  $\mathfrak{D}(A)$  given by the closure in the graph norm of the smooth functions which satisfy  $\partial f_1/\partial n + \gamma f_2 + \sigma f_1|_{\partial\Omega} = 0$ . We remark for future reference that the adjoint semigroup,  $T^*(t)$ , has generator