

Uniqueness of the Cauchy Problem for a Class of Partial Differential Equations with Double Characteristics

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In [11], Trèves pointed out the relation of the theory of local solvability of pseudo-differential equations to the question of uniqueness of the Cauchy problem. Uniqueness of the Cauchy problem $P(x, D)u = f$ in $\phi(x) < 0$, with initial data given on the surface $\phi(x) = 0$, is a consequence of an a priori estimate of the form

$$\int |u|^2 e^{\tau\phi(x)} dx \leq \frac{C}{\tau} \int |Pu|^2 e^{\tau\phi(x)} dx$$

for all $u \in C_0^\infty(\Omega)$ and τ sufficiently large. Such a weighted estimate for P follows from a subelliptic-like estimate for the associated operator $P_\phi(x, D_x, D_t) = P(x, D_x + i|D_t|\phi'(x))$ in x and an auxiliary variable t ($\phi'(x) = \text{grad}_x \phi(x)$). On the basis of results on local solvability and subellipticity of pseudo-differential operators of principal type, Trèves made several conjectures concerning uniqueness of the Cauchy problem. The $\frac{1}{2}$ -subelliptic estimates corresponded to the weighted estimates in Hörmander's book [6]. Trèves' conjecture corresponding to subelliptic estimates for partial differential operators has been proved in [8] for operators whose principal parts are real.

The object of this paper is to prove uniqueness of the Cauchy problem for a class of partial differential equations with double characteristics. Our starting point is the recent local solvability results of Grušin [5], Boutet de Monvel and Trèves [1], [2], and Sjöstrand [9] for operators with double characteristics on non-involutive submanifolds. What is most striking in these results is the delicate influence of the lower order terms on local solvability and hypoellipticity. Local solvability and hypoellipticity hold only if the subprincipal symbol of the operator in question does not take on a discrete set of values. Trèves [12] has given an example where the same discreteness phenomena occurs in the uniqueness of the Cauchy problem. Our theorem generalizes Trèves' example and shows how it is related to local solvability theory. Our method of proof is similar to both [1] and [9]. As in [9] we use Fourier integral operators to reduce