

Adams Type Spectral Sequences and Stable Homotopy Modules

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§1. Introduction. One of the most interesting conjectures in “global” homotopy theory is Freyd’s generating hypothesis (abb. FGH) [8]. There are various interesting consequences and equivalent propositions. The main object of this paper is to study some of these propositions via the Adams type spectral sequence for stable homotopy (abb. ATSS). We will summarize the main applications in this section; and the results on the Adams type spectral sequences in the next section.

First we recall that *Freyd’s generating hypotheses* (FGH) asserts that

FGH. *If $f : X \rightarrow Y$ is a map of two finite CW-complexes and f induces the zero map on stable homotopy modules, then f is stably null homotopic.*

This conjecture has a very interesting implication on injective modules (over the stable homotopy ring π_* of spheres). According to [8, Theorem 9.9], FGH is equivalent to the following statement:

FGH 1. *If X is a finite CW-complex and is a torsion space (i.e., the identity map is of finite order), then $\pi_*(X)$ is π_* -injective.*

We will prove a converse of this statement *without* FGH.

1.1 Theorem. *If X is a finite CW-complex and $\pi_*(X)$ is π_* -injective, then X is a torsion space. In particular, π_* is not self-injective. (See 3.4.)*

Another interesting consequence of FGH is [8, Proposition 9, 4]:

FGH 2. *There is no minimal ideal in π_* i.e., for each $\alpha \in \pi_*$, there is some β of positive degree such that $\alpha\beta \neq 0$.*

We will prove a higher order version of FGH 2.

1.2 Theorem. *Let Y be a CW-complex of finite type. Then $\pi_*(Y)$ has no higher order simple submodules. In particular, π_* has no higher order minimal ideals. (See 4.2.)*

An interesting corollary of this theorem is a result of J. Cohen [6].