

Nonlinear Ordinary and Functional Sturm-Liouville Problems

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1. Introduction. In this paper we study the equation

$$(1) \quad L_\lambda u = \lambda F(\lambda, u), \quad u(0) = u(1) = 0,$$

where L_λ is a Sturm-Liouville operator depending on the parameter λ , and F is some nonlinear operator. We obtain results covering the existence of continua of positive solutions (λ, u) of (1). By imposing various growth conditions on F we obtain results about the behavior of these continua for large and small values of λ and $\|u\|$. These results are related to work done by G. H. Pimbley, P. H. Rabinowitz, R. E. L. Turner and others (see *e.g.* [6], [7], [8], [9]). When dealing with nonlinear problems of this type one usually assumes one of two conditions. One condition is that F is strictly positive, in which case $(\lambda, 0)$ is not a solution for $\lambda = 1$. We make a similar assumption in section 2. An alternate condition is that $F(\lambda, \cdot)$ can be linearized at $u = 0$. Here, in section 3, we examine instead, the case in which $F(\lambda, u) \sim u^\beta$, $\beta \neq 1$, for small u . Problems with nonlinearities of this type have been studied by Z. Nehari, G. B. Gustafson and K. Schmitt (see *e.g.* [5], [1]).

Our results do ultimately depend on the Leray-Schauder theory, however we do not make explicit use of it.

Many of our results carry through under more general boundary conditions and more general conditions on F .

2. Preliminaries.

2.1 Notation. Consider the two point boundary value problem

$$(2) \quad \begin{cases} L_\lambda u \equiv -(p(\lambda, x)u')' + q(\lambda, x)u = \lambda F(\lambda, u), \\ u(0) = u(1) = 0, \end{cases}$$

where p is a positive continuous function on $[0, \infty) \times [0, 1]$,

$$p'(\lambda, x) \equiv \partial p(\lambda, x)/\partial \lambda$$

is continuous, q is a continuous nonnegative function on $[0, \infty) \times [0, 1]$, and F is a continuous, bounded (*i.e.*, takes bounded sets to bounded sets) map