

An Operator Valued Yeh-Wiener Integral, and a Wiener Integral Equation

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§1 **Introduction.** In a series of previous papers¹⁾, the authors have considered the operator valued integral in function space $I_\lambda(F) \equiv I_{\lambda, [a, b]}(F)$ which takes the function ψ into the function $I_\lambda(F)\psi$ whose value at ξ is

$$(1.1) \quad (I_\lambda(F)\psi)(\xi) \equiv \int_{C_1[a, b]} F(\lambda^{-1/2}x + \xi)\psi(\lambda^{-1/2}x(b) + \xi) dx.$$

Here F is a real or complex valued functional defined on the space of all continuous functions on $[a, b]$, the integral is Wiener's integral over the function space $C_1[a, b] = \{x(\cdot) | x \text{ continuous on } [a, b], x(a) = 0\}$, ψ is a real or complex valued function defined almost everywhere on $(-\infty, \infty)$, ξ is a real number and λ is a positive number. The existence and properties of $I_\lambda(F)$ have been established for various classes of functionals F and functions ψ . In particular if

$$(1.2) \quad F(x) = \exp \left\{ \int_a^b \theta(s, x(s)) ds \right\}$$

and $\theta(s, u)$ is bounded and continuous almost everywhere on $[a, b] \times (-\infty, \infty)$, the operator $I_\lambda(F)$ takes L_2 into L_2 . Moreover, if

$$(1.3) \quad F_t(x) = \exp \left\{ \int_t^b \theta(s, x(s)) ds \right\}$$

and

$$(1.4) \quad G(t, \xi, \lambda) = (I_{\lambda, [t, b]}(F_t)\psi)(\xi),$$

then G satisfies the integral equation

$$(1.5) \quad G(t, \xi, \lambda) = \left[\frac{\lambda}{2\pi(b-t)} \right]^{1/2} \int_{-\infty}^{\infty} \psi(v) \exp \left(\frac{-\lambda(v-\xi)^2}{2(b-t)} \right) dv \\ + \left(\frac{\lambda}{2\pi} \right)^{1/2} \int_t^b (s-t)^{-1/2} \int_{-\infty}^{\infty} \theta(s, v) G(s, v, \lambda) \exp \left(\frac{-\lambda(v-\xi)^2}{2(s-t)} \right) dv ds$$

on $[a, b] \times (-\infty, \infty)$. In the case where θ is sufficiently smooth, this equation