

Positive Solutions of Convex Nonlinear Eigenvalue Problems

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1. Introduction. Let $\Omega \subset \mathbf{R}^N$ be a bounded domain of class $C^{2+\mu}$ for some $\mu \in (0, 1)$. In this paper we study nonlinear elliptic eigenvalue problems of the form

$$(1) \quad \begin{aligned} Lx &= \lambda F(x) \quad \text{in } \Omega, \\ Bx &= 0 \quad \text{on } \partial\Omega, \end{aligned}$$

where L is a second order strongly uniformly elliptic differential operator, B is a first order boundary operator such that the pair (L, B) satisfies the strong maximum principle and Schauder *a priori* estimates, and F is the Nemytskii operator of some function $\phi : \bar{\Omega} \times \mathbf{R}_+ \rightarrow \mathbf{R}_+$, that is, for every function $x : \bar{\Omega} \rightarrow \mathbf{R}$, $F(x)$ is the function defined on $\bar{\Omega}$ by

$$F(x)(t) = \phi(t, x(t)), \quad t \in \bar{\Omega}.$$

We suppose that for every $t \in \bar{\Omega}$, the function $\phi(t, \cdot) : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ is increasing, convex, and asymptotically linear.

Problems of this type have recently been studied by Keener and Keller [6] and the authors [3; 10].

By imposing an additional hypothesis on the behaviour of $\phi(t, \xi)$ for large values of ξ and assuming that $\phi(t, 0) > 0$, Keener and Keller showed that there exists a positive number λ^* such that for every $\lambda \in (0, \lambda^*]$, problem (1) has a minimal positive solution and no solution if $\lambda > \lambda^*$. In addition there exists $\delta \in (0, \lambda^*]$ such that for every $\lambda \in (\lambda^* - \delta, \lambda^*)$, the boundary value problem (1) has at least two solutions, whereas for $\lambda = \lambda^*$, the minimal solution is the only solution.

In [3] the first author proved that this result remains true without the hypotheses that $\phi(t, \cdot)$ be increasing and convex. Moreover he showed that in the whole interval $(\lambda_\infty, \lambda^*)$, problem (1) has at least two solutions, where λ_∞ is the principal eigenvalue of the "linearized equation at infinity".

In this paper it will be shown that under the hypotheses used by Keener and Keller, $(\lambda_\infty, \lambda^*)$ is precisely the λ -set for which the boundary value problem (1) has at least two positive solutions. In addition we shall completely charac-