

# *A Note on Shocks in Nonlinear Similarity Fields*

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**1. Introduction.** Similarity analysis, which reduces a second order partial differential equation to an ordinary one via a suitable change of variables has played an important role in many problems of mathematical physics.

Most similarity solutions have been found ad hoc; recently methods for systematic generation of similarity variables and equations have been proposed and applied to a host of problems [1, 2]. Unlike most treatments of partial differential equations, similarity analysis applies to both linear and nonlinear situations and does not depend on the type of the partial differential operator.

Similarity solutions are restricted in that they do not account for arbitrary initial and boundary conditions; however they may be chosen to have a required behavior at certain points or regions (*e.g.* at infinity) so that they faithfully represent the behavior of the field in some asymptotic sense, or at specific regions of interest. Similarity analysis has been most helpful in treating nonlinear heat conduction, boundary layers, explosions and plasmadynamic problems.

Whenever the nonlinear system is of a hyperbolic type, as is the case in all nonlinear wave propagations (in solids, liquids, gases and plasmas), or mixed, *e.g.* transonic or transcritical fluidynamic flows, shocks may be present in the field, be they either generated by external (initial-boundary) conditions or created inside the field.

The question then arises, whether these shocks, mathematically expressed as lines of discontinuity in the dependent (physical) variables across which appropriate jump conditions must be satisfied, will comply with the similarity assumption, and will occur along similarity lines across which the correct jump conditions will be expressible in terms of similarity variables only. This came up in the similarity analysis of explosions, shock propagation, transonic and hypersonic flows, and was treated in each case in an ad-hoc manner [4, 5]. In each case it was possible to fit shocks into the similarity field, and the question is whether it has been simple luck (or magic), or an expression of an inherent property of the mathematical system.

In this note similarity shocks are derived for a system of two quasi-linear,