

Diffusions with Second Order Boundary Conditions, Part II

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This is a continuation of a paper under the same title and by the same author which appeared in the April 1976 issue of this Journal. The notation will be the same.

6. The boundary process. We now restrict our attention to the case when \tilde{G} is a half-space, *i.e.* $G = \{x \in \mathbf{R}^{d+1} : x_1 > 0\}$ and $\partial G = \{x \in \mathbf{R}^{d+1} : x_1 = 0\}$. For this section, we assume a, ρ, γ , and σ satisfy Theorem 5.6 and $b = 0$ and $\beta = 0$. By the results of Stroock-Varadhan [11], if a is assumed to be extended to all of \mathbf{R}^{d+1} , for each $(s, x) \in [0, \infty) \times \mathbf{R}^{d+1}$, there is a unique probability measure $R_{s,x}$ on $C([0, \infty) : \mathbf{R}^{d+1})$ such that

$$R_{s,x}(x(s) = x) = 1,$$

and for each $f \in C_b^{1,2}([0, \infty) \times \mathbf{R}^{d+1})$,

$$f(t, x(t)) - \int_s^t (D_u f + L_u f)(u, x(u)) \, du$$

is a $R_{s,x}$ -martingale where

$$L_u = \sum_{i,j=1}^{d+1} a_{ij} D_{x_i x_j}.$$

Suppose $(s, x) \in [0, \infty) \times G$; define $\tau' = \inf \{t \geq s : x(t) \notin G\}$. Let $P_{s,x}$ be any solution of the submartingale problem of Theorem 5.6 starting at (s, x) for the coefficients a, γ, ρ and σ . By the uniqueness of $R_{s,x}$, we have $P_{s,x} \mid M_{\tau'} = R_{s,x} \mid M_{\tau'}$. We will refer to $R_{s,x} \mid M_{\tau'}$ as the interior process. The main result of this section is that $P_{s,x}$ is uniquely determined by the interior process and a boundary process. We omit most of the details and refer the reader to Section 4 of [12].

Let $\xi : [s, \infty) \times \Omega \rightarrow [0, \infty)$ with $\xi(s) = 0$ be the non-decreasing function corresponding to $P_{s,x}$ given by Theorem 3.2. It follows by the same proof as was given for Corollary 2.3 of [12] that for $x \in \partial G$

$$P_{s,x}(\xi(t) > 0 : t > s) = 1.$$