

Equivariant Function Spaces and Stable Homotopy Theory II

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In a previous paper of the same title [3] the authors studied spaces of equivariant self-equivalences of spheres with free linear actions. This paper provides proofs of some facts alluded to in [3] and its predecessor [2]. The first is an extension of the main theorems in [3] to free cellular actions of finite groups on finite complexes homotopy equivalent to spheres. The second is the existence of an infinite loop space structure on the spaces F_G extending the composition product (mentioned in [2]).

There are several motivations for extending the main theorems of [3] to free cellular finite group actions on (spaces homotopy equivalent to) spheres, among which are the following:

- (i) The extension is strongly suggested by the results of [17] and the existence of free, smooth, but nonlinear actions on spheres (due to R. Lee and T. Petrie).
- (ii) The class of finite groups admitting free actions of the desired type is easily described. It is the class of finite periodic groups by a result of R. Swan [22] and a well-known spectral sequence argument [9].
- (iii) The existence proof for the infinite loop space structures on the spaces F_G of [2, 3] may be formulated to apply to the more general setting (this is done in Section 3).
- (iv) They indicate that proofs for the results of [3] should not require the transversality techniques employed in [19]. One can recover the results of [3] using smooth transversality via Segal's results [19] and consequences of the results in [18]; however, it seems unlikely that similar considerations yield the results of this paper because the corresponding Poincaré transversality theorems are considerably more subtle (compare [14]).

The precise formulation of our results requires some notation. If G is a finite periodic group we may form spaces F_G as in [2, 3] or [17]. Explicitly, let G act freely on the polyhedron $S \simeq S^{n-1}$, let $S^{(k)}$ be the join of k copies of S with itself, and let $F_G(S^{(k)})$ be the space of all equivariant self-maps of $S^{(k)}$. Then $F_G(S^{(k)})$ embeds in $F_G(S^{(k+1)})$ by joining with the identity, and

$$F_G = \text{inj lim } F_G(S^{(k)}).$$