

Nearly Hamiltonian Systems in Nonlinear Mechanics: Averaging and Energy Methods

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1. Introduction. Among the approaches to differential equations, perturbation theory has tended to concentrate on local aspects—continuation of a particular periodic solution or invariant torus as a parameter is varied—while the global approach has dealt principally with “generic” systems, so that its results are seldom applicable to a concrete perturbation problem. This paper is addressed to the global side of perturbation theory. In problems arising from perturbation of Hamiltonian systems in action/angle variables, or from large assemblies of weakly coupled rotators or oscillators, or from weak quasiperiodic forcing, there are an infinite number of possible periodic and quasiperiodic solutions of different frequencies, and it is impossible to build up a global picture by studying the existence or nonexistence of each such orbit separately. We seek to establish nonexistence of invariant tori over large regions of space and to learn how orbits flow between the remaining small open sets. In some cases it is possible to obtain a fairly complete global picture in this way. Our results are vacuous under the hypotheses of the Kolmogorov-Arnol'd-Moser theory ([1], [10]), when invariant tori do in fact exist nearly everywhere in space. The results of this paper were announced in [13]; special cases with two degrees of freedom are contained in [11], [12]. These papers contain examples and motivation not repeated here.

The perturbation problem

$$(1.1) \quad \begin{aligned} \dot{r} &= \epsilon f(r, \theta) \\ \dot{\theta} &= \Omega(r) + \epsilon g(r, \theta) \end{aligned}$$

in which $r = \text{col } (r^1, \dots, r^n) \in \mathbf{R}^n$, $\theta = \text{col } (\theta^1, \dots, \theta^m) \in \mathbf{R}^m$ and f, g are periodic in the components of θ , includes all of the examples mentioned above. The variables r may arise from polar coordinates and be subject to $r^i \geq 0$ (typically for oscillatory motion) or not (typically rotary motion). Regarding θ as a point on the torus T^m and setting $\epsilon = 0$ we obtain for each r an invariant torus supporting a Kronecker flow. These tori are in general foliated by minimal invariant tori of smaller dimension in which the orbits are dense and in fact ergodic; the dimension of the minimal tori depends upon r .