

On the Theta-Operator for Automorphic Forms of Weight 1

CH. POMMERENKE

1. Introduction. Let Γ be a Fuchsian group in the unit disk D , that is a discontinuous group of Moebius transformations of D onto itself. We assume that 0 is not an elliptic fixed point for any element of Γ . A fundamental domain F of Γ is a domain $\subset D$ such that no two points in F are equivalent under Γ and that every point in D is equivalent to some point in F . We assume that

$$(1.1) \quad \text{area } \partial F = 0.$$

The Riemann surface D/Γ is compact if and only if $\bar{F} \subset D$ for some (or every) fundamental domain F .

Let $v(\varphi)$ ($\varphi \in \Gamma$) be a fixed character of Γ , that is a complex-valued function satisfying

$$(1.2) \quad v(\varphi \circ \psi) = v(\varphi)v(\psi), \quad |v(\varphi)| = 1 \quad (\varphi, \psi \in \Gamma).$$

We denote by

$$(1.3) \quad A(\Gamma) = A_1^2(\Gamma, v)$$

the complex Hilbert space of analytic functions $f(z)$ ($z \in D$) with

$$(1.4) \quad v(\varphi)f(\varphi(z))\varphi'(z) = f(z) \quad (\varphi \in \Gamma)$$

(automorphic forms of weight 1 with multiplier v) that satisfy

$$(1.5) \quad \|f\|_{A(\Gamma)} = \left(\frac{1}{\pi} \iint_F |f(z)|^2 d\Omega \right)^{1/2} < \infty$$

where $d\Omega \equiv dx dy$. The *Petersson scalar product* [9]

$$(1.6) \quad \langle f, g \rangle = \frac{1}{\pi} \iint_F f(z)\overline{g(z)} d\Omega \quad (f, g \in A(\Gamma))$$

is independent of the choice of the fundamental domain F because of (1.1). If $v \equiv 1$ then $A(\Gamma)$ corresponds to the space of square integrable analytic differentials on the Riemann surface D/Γ .