Daniell Construction for an H^*-Algebra Valued Measure

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I. In the present article we shall show that the Daniell construction of the extension of an integral is valid also for the case of an H^*-algebra valued positive linear mapping. For the case when the domain of this mapping consists of the class of continuous functions it produces a generalization of Riesz's theorem together with the implication that in this case each positive linear mapping into a trace class is positive definite. The author finds this fact rather remarkable. It seems that an H^*-algebra is a natural object to use (instead of the complex field) in trying to generalize classical theories.

Wright in [10] considered a positive linear mapping into a partially ordered linear space, satisfying a certain condition, which is also valid for H^*-algebras (see Corollary 2 to Lemma 1 below), and obtained a generalization of Riesz's theorem. However, the derived measure there does not have to be regular and the paper [10] does not go beyond the case of continuous functions. In this paper we shall consider an integral defined on a general vector lattice, and the derived measure is regular for the case of topological spaces.

It is appropriate here to mention also a work by I. Kluvanek [4]. His integral is defined on a vector lattice of functions and its range is a subset of a general (unordered) Banach space. The results in [4] are rather interesting but could not really be considered as a generalization of Daniell theory. He also has a generalization of Riesz' theorem for the case of continuous functions (under certain additional assumptions about the integral).

II. We first establish some general lemmas. Let A be a proper H^*-algebra [1] with $(\ ,\ )$ denoting the scalar product and let $\tau A = \{xy \mid x, y \in A\}$ be its trace-class [8]. There is a natural partial ordering $\leqq$ on both A and τA : $0 \leqq x$ if and only if $0 \leqq (xa, a)$ for all $a \in A$ and $x \leqq y$ if $(xa, a) \leqq (ya, a)$ for all $a \in A$. One defines the upper and lower bounds of a subset of A in the usual way; also, it makes sense to talk about lub and glb of a subset E of τA. For example, $a = \text{lub } E$ if a is an upper bound of E and $a \leqq a'$ if a' is any other upper bound of E.

Indiana University Mathematics Journal, ©, Vol. 25, No. 9 (1976)