

Construction of the Scattering Operator for Abstract Nonlinear Wave Equations

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I. Introduction. In the paper, "Scattering for a non-linear wave equation" [5], Morawetz and Strauss construct a scattering operator for the equation

$$(1) \quad u_{tt} - \Delta u + m^2 u = -u^3$$

in terms of the solutions of the related linear equation

$$(2) \quad u_{tt} - \Delta u + m^2 u = 0.$$

Basically, their argument has two parts. First, techniques from partial differential equations are used to prove *a priori* estimates for the decay of solutions of (1) and (2). These proofs make delicate use of energy inequalities and an explicit expression for the Riemann function of (2). Thus, the arguments are quite special to the equations (1) and (2) although fairly general terms on the right hand side of (1) can also be treated.

The second part of the Morawetz-Strauss construction is a functional analysis argument which uses these *a priori* estimates to prove the existence of the scattering operator. This part of the argument is not special to the equations (1) and (2) but, as we show in this paper, can be generalized to an abstract setting. Essentially the theorem we prove says the following: let A be a self-adjoint operator and J a non-linear mapping on a Hilbert space \mathcal{H} . Suppose that we have *a priori* estimates of a certain form on $e^{-itA}\phi_0$ and on the solutions of the non-linear integral equation

$$(3) \quad \phi(t) = e^{-itA}\phi_0 + \int_0^t e^{-iA(t-s)} J(\phi(s)) ds.$$

Then, one can construct a scattering operator for (3). The basic outline of construction follows the ideas of Morawetz and Strauss. We present this generalization for two reasons. First, by separating out and generalizing the construction of the scattering operator, this technique can be used for other non-linear wave equations. Secondly, some of the proofs have been simplified and we hope that this will make the beautiful and difficult work of Morawetz and Strauss more accessible.