

Banach Algebras of Conservative Hausdorff Matrices

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Introduction and notations.

Let c denote the Banach space of all complex or real convergent sequences and let c_0 denote the subspace of c consisting of all null sequences. Let $B(c)$ denote the usual Banach space of all bounded linear operators on c . Let $BV[0, 1]$ denote the Banach space of functions of bounded variation over $[0, 1]$ with the usual norm such that if θ is in $BV[0, 1]$, then $\theta(0) = 0$ and $\theta(t + 0) = \theta(t)$ for t in $(0, 1)$. $C[0, 1]$ denotes the Banach space of the continuous functions on $[0, 1]$ with the sup norm. For θ in $BV[0, 1]$, let H_θ denote the Hausdorff matrix $\{h_{nk}\}$ where $h_{nk} = 0$ for $n < k$ and

$$h_{nk} = \binom{n}{k} \int_0^1 t^k (1 - t)^{n-k} d\theta$$

for $0 \leq k \leq n$, $n = 0, 1, 2, 3, \dots$.

It is well known that H_θ is in $B(c)$. For more information about Hausdorff matrices and conservative matrices the reader might find [1] a useful reference. Let $\mathcal{H} = \{H_\theta : \theta \text{ is in } BV[0, 1]\}$ and $\mathfrak{A} = \{H_\theta : \theta \text{ is absolutely continuous in } [0, 1]\}$. \mathcal{H} is known to be a closed subspace of $B(c)$. Let M denote the Cesaro method of order one. It is well known that if $\theta(t) = t$, then $H_\theta = M$. Thus M is in \mathfrak{A} . For any Banach space X , let X^* denote the usual Banach dual of X .

In this paper we show that \mathcal{H} can be identified with $BV[0, 1]$ and \mathfrak{A} with $L^1[0, 1]$, the Banach space of the absolutely integrable functions in $[0, 1]$. It is well known that $\{C[0, 1]\}^*$ is isomorphic to $BV[0, 1]$. Consequently \mathcal{H} can be identified with $\{C[0, 1]\}^*$ and as such \mathcal{H} can be endowed with the usual weak operator topology and the weak* topology. We show that \mathfrak{A} is weak* dense in \mathcal{H} . We also show that \mathfrak{A} is dense in weak operator topology in \mathcal{H} and that it is not dense in the strong operator topology.

Remark 1. It is proved in [5, Theorem 4] that \mathfrak{A} equals the norm closure of the polynomials in M . The closure of \mathfrak{A} in the weak operator topology certainly contains the commutant of M . In view of Corollary 2 of [3], it follows that the commutant of M in Γ , the Banach space of all conservative matrices, is equal