

# *Stability of Solutions of Mildly Nonlinear Elliptic Problems*

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**1. Introduction.** A solution  $u^*$  of the second order mildly nonlinear elliptic boundary value problem

$$(1.1) \quad Lu = F(u) \text{ in } \Omega, \quad B_\sigma u = \phi \text{ on } \partial\Omega$$

is called classical if  $u^* \in C(\bar{\Omega}) \cap C^\sigma(\Omega \cup S) \cap C^2(\Omega)$ , where  $S$  is the smooth part of the boundary and  $\sigma = 0$  or  $\sigma = 1$  according as  $B_\sigma$  is a zeroth or first order derivative boundary operator. We are interested in the existence and stability of classical solutions of (1.1).

Customarily the stability of  $u^*$  is studied by perturbing the right hand sides of (1.1) and examining the behavior of the corresponding solutions, by considering the solutions  $z(x, t; v)$  of the associated parabolic problems

$$(1.2) \quad \begin{aligned} \partial z / \partial t + Lz &= F(z) \quad \text{in} \quad \Omega \times \mathbf{R}^*, & Bz &= \phi \quad \text{on} \quad \partial\Omega \times \mathbf{R}^* \\ z(\cdot, 0) &= v \quad \text{on} \quad \bar{\Omega} \end{aligned}$$

where  $\mathbf{R}^*$  is the set of positive real numbers and  $v$  is close to  $u^*$ , or by determining the locations of the eigenvalues of the linearized eigenvalue problem

$$(1.3) \quad Lv = \mu F'(u^*)v, \quad Bv = \phi.$$

Corresponding to these methods of study,  $u^*$  may be called stable to perturbations, asymptotically stable, or strictly stable. Strict stability, which asserts that the minimal eigenvalue of (1.3) exceeds 1, is the notion most often encountered in the engineering literature, because it ensures that the solution is not at a bifurcation point of the associated nonlinear eigenvalue problem

$$(1.4) \quad Lu = \mu F(u), \quad Bu = \phi.$$

Asymptotic stability is the notion familiar from the classical work of Ljapunov; its application to partial differential equations is at least 25 years old (see [23, pp. 221–223]). The definition we give of stability to perturbations is essentially that introduced by Parter [15] for maximal solutions of (1.1).

Various physical problems lead to equations of the form of (1.1) (see [1], [7]–[9], [18]–[23]), and some effort has been expended to determine solutions