

# *Perturbation of the Laplacian by Coulomb Like Potentials*

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*In memory of J. M. Jauch*

**1. Introduction.** This paper is concerned with the scattering and spectral theory for the Schrödinger operator

$$(1.1) \quad L = -\Delta + \frac{1}{i} \sum_{i=1}^n a_i(x) \frac{\partial}{\partial x_i} - \frac{\alpha}{|x|} + q(x)$$

in  $\mathbf{R}^n$  ( $n \geq 3$ , odd  $n$ ) with  $\alpha \in \mathbf{R}$  and where  $q(x)$  and  $a_i(x)$  are appropriate short-range perturbations of the Coulomb potential  $\alpha/|x|$ . More precisely we suppose that the following conditions hold:

(1.2) The real-valued functions  $a_i(\cdot)$  are continuously differentiable in  $\mathbf{R}^n$  and  $(1 + |x|)^a a_i(x) \in L^{p_i}(\mathbf{R}^n)$  with  $a > n/2$  and  $n < p_i < 2n$ .

(1.3) The complex-valued function  $q(\cdot)$  is a measurable function defined a.e. in  $\mathbf{R}^n$  and we suppose that the function  $Q(x)$  defined though  $q(x) = (1 + |x|)^{-a} Q(x)$  is expressible as a sum  $Q = Q_1 + Q_2$  of  $Q_i \in L^{q_i}(\mathbf{R}^n)$  and  $Q_2 \in L^{q_2}(\mathbf{R}^n)$  where each of  $q_i (i = 1, 2)$  satisfies  $n/2 < q_i < 2n$  and  $2 \leq q_i$ .

(1.4)  $L$  is a formally selfadjoint operator.

In the physical case, the differential operator  $L$  represents the energy operator of an electron in an exterior electromagnetic field, interacting with a fixed source through the Coulomb potential  $\alpha/|x|$  and a short range one.

When  $\alpha = 0$ , the scattering and spectral theory of the operator  $L$  has recently been brought into very satisfactory condition. The most recent and general results in this direction have been obtained by S. Agmon [1], S. T. Kuroda [2] and M. Schechter [3*a, b*]. Moreover, when  $q(x)$  and  $a_i(x)$  are long range perturbations, T. Ikebe and Y. Saito [4] and R. Lavine [5] (in the case  $a_i(\cdot) = 0$ ) have got a very general result concerning the spectrum of the operator  $L$  (see also J. Aguilar and J. M. Combes [6], P. A. Rejto [7] and J. Weidmann [8]). Existence of generalized wave operators has been shown by J. Dollard [9], W. Amrein, P. Martin and B. Misra [10], V. S. Buslaev and V. B. Matveev [11] P. Alshom and T. Kato [12], and K. Zizi ([13], [33]) (see also R. Lavine [5]).