

Hyperbolic and Other Symmetrizable Systems in Regions with Corners and Edges

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Introduction. We shall be concerned with first order linear systems L of partial differential operators in regions with corners and edges. The basic difficulty presented by corner problems as opposed to half-space problems, especially for non symmetric systems, is this: one can often analyze non-characteristic half-space problems with the help of pseudo-differential operators in the tangential variables or, in the constant coefficient case, with the help of a 'tangential' Fourier transform. When there are corners present, these methods tend to break down because of the non-local character of pseudo-differential operators and of the Fourier transform, and because there are too many 'non-tangential' variables. For hyperbolic equations in particular, the approach to the questions of existence and uniqueness via 'tangential symmetrizers' used, *e.g.*, by Calderon [3] for the Cauchy problem and by Kreiss [10] for the mixed problem, runs into serious difficulties in the presence of corners. We shall show, on the other hand, that the Friedrichs-Lax approach [6], [7], via a 'full symmetrizer', does yield positive results for corner problems, especially with strictly hyperbolic or elliptic equations.

We shall restrict our attention to square systems which are either symmetric or are symmetrizable in the sense of Friedrichs and Lax [6]. Specifically, if the leading symbol of L is $ia(x, \xi) = i \sum A_i \xi_i$, then there exists a zero-order pseudo-differential operator $R = r(x, D)$ with symbol r such that ra is symmetric; R is called a symmetrizer for L . We shall study boundary value problems for L in a region Ω of \mathbf{R}^m having a uniformly non-characteristic and piecewise smooth boundary Γ . Our starting point will be a quadratic identity satisfied by functions $u \in C_0^\infty(\bar{\Omega})$. If u is extended to vanish outside of $\bar{\Omega}$, then, as Friedrichs and Lax show in the case of a smooth Γ , integration by parts of the form $\text{Re } (u, Ru)$ leads formally to a quadratic identity

$$(0.1) \quad 2 \text{Re } (u, Rf) = (u, Hu) + 2 \text{Re } (R^*u, A_n u)_\Gamma .$$

Here f is the restriction of Lu to Ω , A_n is the coefficient in L of outward normal differentiation along Γ , and H is a zero-order pseudo-differential operator. R^*u will in general be discontinuous across Γ , and the restriction $R^*u|_\Gamma$ of