

Uniqueness of Nonparametric Area Minimizing Currents

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H. Federer [3, 5.4.18] proved that the oriented graph of a class 2 function on an open subset U of \mathbf{R}^m is an absolutely area minimizing current in $U \times \mathbf{R}$ if and only if the function satisfies the minimal surface equation. Suppose U is convex and f is a continuous function on $\text{Clos } U$ whose restriction to U satisfies the minimal surface equation. Here we show (Theorem 2) that, for any $m - 1$ dimensional integral current P whose support lies in the graph of f , whose boundary is zero, and whose projection in \mathbf{R}^m bounds a positive current Q , the m dimensional integral current lying over Q in the graph of f is the unique absolutely area minimizing current in $\mathbf{R}^m \times \mathbf{R}$ having boundary P . The elementary proof involves the closed m form in $U \times \mathbf{R}$ of [3, 5.4.18], obtained by vertically translating the volume form of the graph of f . Such a differential form argument was used, with a divided power of the Kahler form, in establishing that a positively oriented compact subset K of a holomorphic subvariety of \mathbf{C}^m is the unique [8, 3.6] absolutely area minimizing [3, 5.4.19] current having boundary ∂K ; this idea is greatly generalized in [7, 4.10–4.15] where Simon's cone is also treated [7, 6.3]. Our Theorem 1 provides a new proof (Corollary 1) of J. Brothers Sheetting Lemma [1] and leads easily to the well known fact [1, 3.8] that an absolutely area minimizing integral current in Euclidean space is supported by the convex hull of the support of its boundary. From Theorem 2, in case $m = 2$, it follows that any J. Douglas [2] Plateau problem solution of arbitrary topological type, for a rectifiable Jordan curve having convex planar projection, is necessarily the nonparametric disc-type solution. The author wishes to thank J. C. C. Nitsche for suggesting the latter problem.

Herbert Federer has kindly pointed out the formal similarity between his functional isoperimetric inequality in [6, §2] and the following equivalent form of our Theorem 1:

If T is an indecomposable integral current and u is a real-valued class 1 function such that $\vec{T} \lrcorner Du = 0 \ ||T|$ -almost everywhere, then $u \mid \text{spt } T$ is constant.

He also noted that the existence in Theorem 2 of a suitable P , for any given positive integral current Q with support in $\text{Clos } U$, is equivalent by [3, 4.5.9]