

Asymptotic Stability with Rate Conditions, II

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In this paper we study smoothness of invariant families of manifolds and exponential rate conditions for dynamical systems with homoclinic points. These topics are related to an earlier paper [3], but otherwise are independent. We assume that the reader is familiar with [3].

We study smoothness of invariant families in §I. Let Λ be an overflowing invariant manifold, let (Λ, E) be an invariant set with expanding structure, and let $\{W_{1oc}^E(m) : m \in \Lambda\}$ be the invariant family determined by (Λ, E) , as in [3, §I.D]. Each manifold $W_{1oc}^E(m)$ is smooth. $\{W_{1oc}^E(m) : m \in \Lambda\}$ may not be smooth as a family, however, because $W_{1oc}^E(m)$ may not depend smoothly on m . See the example in [3, §I.H]. We prove a smoothness theorem for invariant families, and a closely related smoothness theorem for invariant sub-bundles of vector bundles. Both theorems are expressed in terms of the exponential rates α^* and ρ^* of [3], and an exponential rate τ^* . As an application, we prove a smoothness theorem for the unique asymptotic phase defined in [3]. We also prove some smoothness theorems for Anosov systems.

§II is a collection of examples of exponential rate conditions for dynamical systems with homoclinic points. Our main conclusion is that exponential rates of ratios, as introduced in [2], are finer than ratios of exponential rates. The invariant manifold construction of Hadamard is applicable to problems in which rate conditions are defined in terms of growth rates of ratios, as in [2] and [3]. The invariant manifold construction of Perron does not seem to be applicable to such problems, so we conclude that Hadamard's construction is finer than Perron's construction. Hadamard's construction is finer than any construction based on separation of spectra of linear operators.

Selgrade associated growth rate intervals with a Morse decomposition of a vector bundle flow [10]. He proved that the growth rate intervals are disjoint if the flow is defined over a connected chain recurrent base. One of our examples shows that the growth rate intervals may overlap if the base is not connected.

Mañé has announced a converse perturbation theorem for invariant manifolds [8]. One of our examples shows that normal hyperbolicity must be defined in terms of growth rates of ratios, in order for Mañé's result to hold.

Our final example shows that one of the hypotheses of the metric characterization of expanding families [3, Theorem 3] can not be omitted.