

Solutions in the Large for the Equations of Nonisentropic Gas Dynamics

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1. Introduction. We consider the nonisentropic gas equations in Lagrangian coordinates (see, e.g. [1])

$$(1.1) \quad \left\{ \begin{array}{ll} \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 & \text{(conservation of momentum)} \\ \frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = 0 & \text{(conservation of mass)} \\ \frac{\partial E}{\partial t} + \frac{\partial(pu)}{\partial x} = 0 & \text{(conservation of energy)} \\ p = p(v, e) = p(v, s) & \text{(equation of state)} \end{array} \right.$$

where $t \geq 0$, $-\infty < x < \infty$, and $u, v > 0$, $p > 0$, $e, s > 0$ are the velocity, specific volume, pressure, internal energy and entropy, $E = u^2/2 + e$ is the total energy and $\rho = 1/v$ the density.

The problem to be solved is the Cauchy problem (1.1) with initial data

$$(1.2) \quad (u(x, 0), v(x, 0), E(x, 0)) = (u_0(x), v_0(x), E_0(x)), \quad -\infty < x < \infty.$$

We seek the *weak* solution $(u, v, E)(x, t)$ of (1.1) & (1.2), i.e. $(u, v, E)(x, t)$ satisfies the integral equations

$$(1.3) \quad \left\{ \begin{array}{l} \iint_{t>0} (uf_t + pf_x) dx dt + \int_{t=0} u_0(x)f(x, 0) dx = 0, \\ \iint_{t>0} (vg_t - ug_x) dx dt + \int_{t=0} v_0(x)g(x, 0) dx = 0, \\ \iint_{t>0} (Eh_t + puh_x) dx dt + \int_{t=0} E_0(x)h(x, 0) dx = 0, \end{array} \right.$$

for all smooth functions f, g and h with compact supports.

Our existence theorem is proved under the assumption that the gas is poly-