

# *Boundary of a Fuzzy Set*

RICHARD H. WARREN

**1. Introduction.** In this paper we establish a boundary for a fuzzy set in a fuzzy topological space with the following properties:

- 1) the boundary is closed,
- 2) the closure is the supremum of the interior and boundary,
- 3) the boundary reduces to the usual topological boundary when all fuzzy sets are crisp,

4) the boundary operator is an equivalent way of defining a fuzzy topology. None of the many equivalent definitions of the boundary in general topology were found to have these properties in a fuzzy topological setting. Consequently, the definition of the fuzzy boundary is not recognizable from general topology.

The boundary, as defined in this paper, lacks some of the properties that one might like a boundary to have, such as:

- 5) the boundary of a fuzzy set is identical to the boundary of the complement of the set,
- 6) if a fuzzy set is closed (or open), then the interior of the boundary is empty,
- 7) if a fuzzy set is both open and closed, then the boundary is empty.

However, property 5 can hold only if one gives up property 2, as we shall show in Example 3.1. Perhaps, Theorem 3.6(iv) is an adequate substitute for property 7. In the restrictive case of property 3, then properties 5, 6 and 7 are valid.

The importance of the boundary in a fuzzy topological space has been established in [4] and [7] where the boundary has a significant role in a generalization of modern control theory. This paper is a foundation stone for that work.

Saaty [5, pp. 53, 54] notes that the property which characterizes a fuzzy set may tend to fade away at the boundary and thus, analysis of the property at the boundary can be more difficult. By giving precise mathematical meaning to the boundary, we overcome one of the underlying features of this difficulty.

Bellman [1] encourages the development of fuzzy set theory because it seems to have promising applications to the less quantitative problems of society. This paper continues the development of fuzzy topological spaces begun in [2], [3], and [6] which go back to the fundamental concepts of fuzzy sets introduced by Zadeh [8]. General topology can be regarded as a special case of fuzzy topology where all fuzzy sets are crisp.