

Acoustic Scattering from Locally Reacting Surfaces

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1. Introduction. When a sound wave in a homogeneous medium interacts with an obstacle, the scattered wave motion depends on the nature of the surface of the obstacle. Surfaces are considered in theoretical acoustics whose *specific acoustic impedance*, the ratio between the pressure of the wave and the normal velocity at the surface, is dependent on the frequency of the incident wave. We will assume here that the surface is nonporous and *locally reacting* in the sense that wave motion along the surface is negligible. In this paper we develop a theory of scattering from such surfaces following a general approach of Lax and Phillips [6] for dissipative hyperbolic systems. We suppose that a small part of the surface reacts to the excess pressure due to the wave like a resistive harmonic oscillator (this model is used for example in [11]; see p. 263).

This boundary condition differs from others for the acoustic equation in that it does not have the so-called coercive property. However, with certain assumptions on the parameters occurring in the boundary behavior, it is possible to find a special class of initial data, dense in the energy norm, whose solutions form pre-compact sets with respect to local energy. As a consequence we establish the local decay of arbitrary solutions of finite energy. The scattering operator for this problem has the usual representation in terms of the scattering matrix S , an operator-valued function on \mathbf{R} which is the limit of an analytic function on the lower half-plane. When one of the standard boundary conditions is used for the acoustic equation, S has a meromorphic extension to the entire plane; in the present case, the extended S can have essential singularities at points which are natural (complex) frequencies of the boundary motion. In fact, if the parameters in the boundary condition are constant, these frequencies are accumulation points of poles of S . This behavior corresponds to certain unusual spectral properties for this boundary condition in bounded domains, previously described in [1].

The model we consider is as follows. Let G be an exterior domain in \mathbf{R}^3 in which a homogeneous fluid is at rest except for sound waves. If $\phi(x, t)$ is the velocity potential (*i.e.*, $-\nabla\phi$ is the particle velocity), then ϕ satisfies the acoustic wave equation