

# Commutants of Generalized Integrations on a Space of Analytic Functions

R. M. CROWNOVER & R. C. HANSEN

**1. Introduction.** We study the commutants of *generalized integration* operators on the space of analytic functions on the open unit disk in the plane. A *generalized integration* is a continuous operator that maps the space of analytic functions onto the subspace of those functions that vanish at the origin and that have strictly lower triangular matrix representations. Special attention is given to weighted shifts, multiplications, and Volterra operators that are generalized integrations.

Let  $\mathbf{C}$  be the complex plane and let  $\mathcal{A}$  be the space of analytic functions on  $\{z : z \in \mathbf{C} \text{ and } |z| < 1\}$ , endowed with the topology of uniform convergence on compact sets. Let  $\mathcal{L}(\mathcal{A})$  denote the space of continuous linear operators from  $\mathcal{A}$  into itself. With the *strong operator topology*, defined at the zero operator by the sets

$$N(0; f, K, \epsilon) = \{A : A \in \mathcal{L}(\mathcal{A}) \text{ and } |Af(z)| < \epsilon \text{ for } z \in K\},$$

where  $f \in \mathcal{A}$ ,  $K$  is compact and  $\epsilon > 0$ , the space  $\mathcal{L}(\mathcal{A})$  is a locally convex topological space with a multiplication (operator composition) that is continuous in each factor separately. Thus  $\mathcal{L}(\mathcal{A})$  with the strong operator topology is a topological algebra [9; Chapter II].

We present here a description of the *commutants* of (operators that commute with) certain generalized integration operators which are described below. Included as special cases of these generalized integrations are many *weighted shifts* (studied by N. I. Nagnibida [7]), *multiplications* (see K. M. Fishman [4] and Nagnibida [8]), and *Volterra integration operators*.

Our main results on commutants concern the maximality of the principal ideals generated by generalized integrations and the resulting formal power series representations of elements in the commutants. Linear space isomorphisms of commutants onto  $\mathcal{A}$  are investigated for weighted shifts and multiplications. The structure of the carrier space of the commutant of a Volterra integration operator is considered in the last section.

Every  $A \in \mathcal{L}(\mathcal{A})$  is known to have a matrix representation  $(a_{i,j})_{i,j=0}^{\infty}$  such that if  $f(z) = \sum_{n=0}^{\infty} b_n z^n \in \mathcal{A}$ , then