

# *$\mathfrak{N}_0'$ -Spaces and a Function Space Theorem of R. Pol*

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**1. Introduction.** The following interesting theorem was recently obtained by R. Pol [8, Theorem 1 (iii)]: *If  $X$  is a metrizable space with separable derived set<sup>(1)</sup>  $X'$ , and if  $K$  is compact metrizable, then the function space<sup>(2)</sup>  $K^X$  is Lindelöf.* Pol's proof of this result is rather lengthy, and the first purpose of this note is to give a proof which is somewhat shorter and simpler. Like Pol, we shall actually prove a bit more, namely that  $K^X$  belongs to a class of Lindelöf spaces which is preserved by countable products.<sup>(3)</sup> Although our class and Pol's are defined differently, it can be shown that they actually coincide.

We now state our theorem. For the definition of Lindelöf  $\Sigma$ -spaces, see section 2.

**Theorem 1.1.** *If  $X$  is a metrizable space with separable  $X'$ , and if  $K$  is compact metrizable, then  $K^X$  is a Lindelöf  $\Sigma$ -space.*

Let us now generalize Theorem 1.1 to non-metrizable spaces  $X$ . It was observed by Pol [8] that, if  $X$  is any space and  $Y$  is a compact-covering<sup>(4)</sup> quotient image of  $X$ , then  $K^Y$  is homeomorphic to a closed subset of  $K^X$ . Since Lindelöf  $\Sigma$ -spaces are preserved by closed subsets, it follows that Theorem 1.1 remains valid if  $X$  is only assumed to be a compact-covering quotient image of a metrizable space  $M$  with separable  $M'$ . As we shall see in Theorem 4.4, this is equivalent, for regular  $X$ , to  $X$  being a  $k$ -space and an  $\mathfrak{N}_0'$ -space, where  $\mathfrak{N}_0'$ -spaces are defined as follows.

**Definition 1.2.** A space  $X$  is an  $\mathfrak{N}_0'$ -space if there is a countable collection  $\mathcal{O}$  of subsets of  $X$  such that, if  $K \subset U$  with  $K$  compact and  $U$  open in  $X$ , then  $K \subset P \cup F \subset U$  for some  $P \in \mathcal{O}$  and finite  $F \subset X - X'$ .

Observe that, if one can always take  $F$  empty in the above definition, one obtains an  $\mathfrak{N}_0$ -space in the sense of [3],<sup>(5)</sup> which explains the terminology. Clearly every  $\mathfrak{N}_0$ -space is an  $\mathfrak{N}_0'$ -space, and if  $X$  is an  $\mathfrak{N}_0'$ -space then  $X'$  is an  $\mathfrak{N}_0$ -space.

It is easily seen that every metrizable space  $X$  (more generally, every space  $X$  with a point-countable base) with separable  $X'$  is an  $\mathfrak{N}_0'$ -space. In fact, if  $\mathfrak{B}$