

Isometries of the Nevanlinna Class

KENNETH STEPHENSON

1. Introduction.

1.1. The motivation for this work is the recent characterization of linear isometries of the H^p -spaces associated with the unit polydisc U^n and the unit ball B_n in the space of n complex variables. This was done in a sequence of papers by Forelli [6] and [7], Schneider [12], and finally Rudin [11]. The H^p -spaces lie in the Smirnov class N_* which in turn is contained in the Nevanlinna class N of functions of bounded characteristic. A natural metric on N has been studied by Davis [2] and [3], Shapiro and Shields [13], and Roberts [9]. We will consider isometries of N and N_* in this metric.

Our main result is the complete characterization of isometries of the Smirnov class, given in Section Two. We then consider isometries of the Nevanlinna class, with some general n -variable results in Section Three. Sections Four and Five are devoted to the one-variable case, where the factorization theorem leads to many interesting questions on compositions of inner functions. The one-variable results on isometries of N are summarized at the end of Section Five. The paper ends with some open questions.

Basic definitions and facts about the Nevanlinna class are provided in Sections 1.2 and 1.3. For details the reader is referred to [2], [4], [10], and [13]. In Section 1.4, the characterization of isometries of the H^p -spaces is quoted from Rudin [11]. Duren [4] is a good reference for the properties of inner functions needed in Section 5.

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1.2. Definitions. \mathbf{C}^n , $n \geq 1$, denotes the space of n complex variables, $z = (z_1, \dots, z_n)$. The domains we consider are:

(a) The unit polydisc $U^n = \{z \in \mathbf{C}^n : |z_i| < 1, 1 \leq i \leq n\}$ with distinguished boundary $T^n = \{z \in \mathbf{C}^n : |z_i| = 1, 1 \leq i \leq n\}$.

(b) The unit ball

$$B_n = \left\{ z \in \mathbf{C}^n : \sum_1^n |z_i|^2 < 1 \right\}$$

with boundary