

Full Algebras of Operator Fields Trivial Except at One Point

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Introduction. While the structure of commutative C^* -algebras is well-understood and much work has been done on general non-commutative C^* -algebras, there are many specific C^* -algebras (such as the group C^* -algebras of non-commutative non-compact groups) about whose structure little is known. An outstanding example is the group C^* -algebra of the Heisenberg group. Given a specific C^* -algebra A , one method of studying its structure is to express it as a C^* -algebra of “continuous” operator-valued functions on a locally compact Hausdorff space T . There is a canonical way to do this (see [4]), which we do not need to discuss in detail. Suffice it to say that the “continuity” of the operator-valued functions x on T corresponding to elements of A is expressed by saying that the set of all such x forms what is called a full algebra of operator fields (see Section I for definition). It is with full algebras of operator fields that this paper is concerned.

In view of this connection with the structure theory of general C^* -algebras, one would like to be able to classify all full algebras of operator fields on a given locally compact Hausdorff space and with arbitrary given fibre algebras. An important first step in this program is the Stone–Weierstrass–Glimm Theorem ([4], Theorem 1.4), which reduces the general problem to that of the classification of maximal ones. Even this, however, is far too difficult in general. In this paper, we shall take a special case, and try to classify those maximal full algebras of operator fields which are “trivial except at one point” in the following sense: if t_0 is a non-isolated point in T , and if all the fibre algebras except A_{t_0} are the same, say D , a maximal full algebra of operator fields A is called trivial except at t_0 if, for each x in A , $x|_{T - \{t_0\}}$ is continuous from $T - \{t_0\}$ to D .

This paper is divided into four sections. In the first section, some definitions and general properties of full algebras of operator fields are given. Section II is devoted to special situations. In examples 1, 2 and 3, we completely classify all maximal full algebras of operator fields trivial except at one point on certain given spaces, with certain given uncomplicated fibre algebras. Using Theorem 4 of [8], we prove in section III that the group C^* -algebra of the Heisenberg