

A New Result of Duality for Spectral Decompositions

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Introduction. In this paper we shall investigate the behavior of the spectral decompositions with respect to the duality. We have in view some variants of spectral decompositions in the sense of C. Foiaș ([4], [2]).

In ([5]), we have proved that the dual of a 2-decomposable operator is 2-decomposable. In ([11]), F.-H. Vasilescu has defined the decomposable operators with almost localized spectrum and has proved that the dual of such an operator is of the same type. Recently, the author has proved ([7]) that the dual of a 2-decomposable operator is actually decomposable. This result is related, in particular, to the problem if any 2-decomposable operator is decomposable ([10]). By using the simple arguments of topological dimension theory, E. Albrecht and F.-H. Vasilescu have proved in ([1]) that any 3-decomposable operator is decomposable. As a simple corollary of our result, one obtains that, on a reflexive Banach space, any 2-decomposable operator is decomposable.

We shall prove, in the present paper, that the dual of a 2-decomposable operator is a decomposable operator with almost localized spectrum. Thus we improve both our preceding result from ([7]) and the result of F.-H. Vasilescu from ([11]).

Our present result is related to the question raised by I. Colojoară and C. Foiaș ([2], p. 217) whether the restriction of a decomposable operator to a spectral maximal space is again decomposable. Although we were not able to answer this difficult problem, a consequence of our result is that every decomposable operator, on a reflexive Banach space, has almost localized spectrum.

The paper is composed of four sections. The main result is in Section 4. For its proof we use some results of a general nature which may have also an intrinsic importance. Thus, in Section 3 we prove a general theorem concerning the extension of sequences of analytic functions (Theorem 2) and, in Section 2, a decomposition theorem for linear continuous functionals (Theorem 1).

Preliminaries. Let X be a complex Banach space and $L(X)$ be the algebra of all linear continuous operators on X . An operator $T \in L(X)$ is said to have the single-valued extension property if there does not exist any X -valued analytic