

*R-Operators II.*  
*On the Approximation of Certain*  
*Operator-Valued Analytic Functions and*  
*the Hermitian Moment Problem*

FRANCIS J. NARCOWICH

**I. Introduction.**

**Origins and statement of the problem.** Let  $\mathfrak{H}$  be a complex separable Hilbert space and let  $B(\mathfrak{H})$  be the set of all bounded operators on  $\mathfrak{H}$ . An  $R$ -operator,  $T(z)$ , is a  $B(\mathfrak{H})$ -valued function which is analytic in the upper and lower half-planes and which satisfies

$$\operatorname{Im} z \cdot \operatorname{Im} T(z) \geq 0.$$

The set of all  $R$ -operators will be denoted by  $\mathfrak{W}$ .  $R$ -operators are characterized by the

**Representation Theorem 1.1.** *If  $T(z) \in \mathfrak{W}$ , then there is a non-negative operator  $A$ , an Hermitian operator  $B$  and an increasing strongly additive, positive, operator-valued measure,  $\nu(\cdot)$ , defined on the Borel sets of  $(-\infty, \infty)$  such that*

$$(1.1) \quad T(z) = Az + B + \int_{-\infty}^{\infty} \frac{tz + 1}{t - z} d\nu(t).$$

*Moreover, if  $T(z)$  is analytic over some interval of the real axis, then  $\nu(t)$  is constant on this interval and  $\operatorname{Im} T = 0$ . Finally,  $\nu(\cdot)$  is uniquely determined by  $T(z)$ .*

*Proof.* See Allen and Narcowich, [3], Theorems 2.1 and 2.3.

$R$ -operators arise in conjunction with various aspects of Quantum Mechanics (cf. Allen and Narcowich, [3] and [11]), where the variable  $z$  is interpreted as energy or as a coupling constant for some interaction. In the latter case, one is often confronted with the following problem: given that  $T(z)$  is an  $R$ -operator which is analytic on some interval of the real axis,  $(-a, 0)$ , as well as in the upper and lower half planes, and given that  $T(z)$  is known to have the asymptotic expansion