

On a Mean Value Inequality

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Introduction. In this paper we discuss a mean value inequality satisfied by functions $u(x, t)$ defined in the half space R_+^{n+1} which are solutions of a differential equation of parabolic type. We then apply this result to extend some recent work of Fabes, Johnson and Neri [5] and Fabes and Neri [7] concerning the characterization of functions which arise as traces of harmonic functions and temperatures in R_+^{n+1} .

The justification for studying the functions $u(x, t)$ lies in the fact that they are a natural substitute for harmonic functions when Laplace's equation is not satisfied and they are closely related to the study of the so called parabolic H^p spaces defined in [2]. In fact the differential equations which we consider are introduced there and the preliminary version of that paper essentially contained Theorem 1.4.

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Notation. We shall introduce the notation as we go along. Here we only remark that the letter c will denote a constant which need not be the same in different occurrences.

1. The mean value inequality.

1.1. We let $\{A_t\}_{t>0}$, $A_{t,s} = A_t A_s$ be a continuous group of affine transformations of R^n leaving the origin fixed and denote its infinitesimal generator by P so that $t(d/dt)A_t = PA_t$. We further assume that $(Px, x) \geq (x, x)$ for $x \in R^n$ and associate to each group A_t a translation invariant distance function $\rho(x)$ defined to be the unique value of t such that $|A_t^{-1}x| = 1$, $\rho(0) = 0$. We have that $\rho(A_t x) = t\rho(x)$ for $t > 0$. To the transpose A_t^* of A_t we associate $\rho^*(x)$ in a similar fashion. As is well known, $\det A_t = \det A_t^* = t^\gamma$, $\gamma = \text{trace } P$.

With T_t and \tilde{T}_t we will denote the operators defined by

$$(T_t f)(x) = f(A_t x), \quad (\tilde{T}_t f)(x) = f(A_t^* x).$$