

Fredholm Properties of a Class of Toeplitz Operators on the Ball

GERARD McDONALD

1. Introduction. Let B denote the unit ball in \mathbb{C}^n ,

$$B = \{z = (z_1, \dots, z_n) \in \mathbb{C}^n : |z| < 1, |z|^2 = |z_1|^2 + \dots + |z_n|^2\},$$

and $L^2(B)$ the space of square-integrable functions on B . (We will consider only the usual Lebesgue measures on B and the unit sphere S . Henceforth n will refer to, or be equal to, the complex dimension of B .) The collection of square-integrable holomorphic functions on B , $H^2(B)$, forms a closed subspace of $L^2(B)$. The orthogonal projection of $L^2(B)$ onto $H^2(B)$ will be written P . The algebra of bounded linear operators on $H^2(B)$ and its norm closed ideal of compact operators will be denoted, respectively, by $\mathfrak{L}(H^2(B))$ and \mathfrak{K} . For φ in $L^\infty(B)$, the Toeplitz operator with symbol φ , T_φ , is defined by

$$T_\varphi f = P(\varphi f), \quad f \in H^2(B).$$

If \mathfrak{S} is a subset of $L^\infty(B)$, the closed subalgebra of $\mathfrak{L}(H^2(B))$ generated by $\{T_\varphi : \varphi \in \mathfrak{S}\}$ will be written $\mathfrak{J}(\mathfrak{S})$.

Denote the space of bounded holomorphic functions on B by $H^\infty(B)$. For f in $H^\infty(B)$ and almost every ζ on S the limit

$$f^*(\zeta) = \lim_{r \rightarrow 1} f(r\zeta)$$

exists. The map $f \rightarrow f^*$ is an isometric isomorphism onto the closed subalgebra of $L^\infty(S)$, $H^\infty(S)$. The spaces

$$H^\infty(B) + C(\bar{B}) = \{f + g : f \in H^\infty(B), g \in C(\bar{B})\}$$

and

$$H^\infty(S) + C(S) = \{f + g : f \in H^\infty(S), g \in C(S)\}$$

are closed subalgebras of $L^\infty(B)$ and $L^\infty(S)$, respectively [7]. The map $\varphi \rightarrow \varphi^*$, $\varphi^*(\zeta) = \lim_{r \rightarrow 1} \varphi(r\zeta)$, is an algebra homomorphism of $H^\infty(B) + C(\bar{B})$ onto $H^\infty(S) + C(S)$.

Toeplitz operators with continuous symbol have been studied by Venugopalakrishna [9] and Coburn [1]. The following theorem can be found in [1].