Evolution Systems Related to Gaussian Markov Processes and an Integral Equation

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Introduction. R. H. Cameron and D. A. Storvick introduced their operator valued function space integral by using the Wiener integral in [6]. J. A. Beekman and R. A. Kallman defined a more general operator valued function space integral in [5] by using the Gaussian Markov expectation. They proved that the Gaussian Markov integral of an appropriate function satisfies an integral equation analogous to the integral equation as in [6]. It is the purpose of this paper to derive the integral equation obtained by J. A. Beekman and R. A. Kallman by using only the properties of evolution systems. We shall use the same notations as in [6], [15].

1. Gaussian Markov processes and evolution systems. Let us consider the function

$$p_\lambda(s, x : t, y) = \left(\frac{\lambda}{2\pi A(s, t)}\right)^{1/2} \exp\left(-\frac{\lambda(y - xv(t)/v(s))^2}{2A(s, t)}\right), \tag{1.1}$$

where λ is a complex number such that $\operatorname{Re}\lambda \geqq 0$, $\lambda \neq 0$ and

$$A(s, t) = u(t)v(t) - u(s)v(t)^2/v(s), \qquad 0 \leqq s \leqq t \leqq T, \tag{1.2}$$

$$u(t) \geqq 0, \qquad v(t) > 0, \qquad 0 \leqq t \leqq T, \tag{1.3}$$

$$u'(t), \qquad v'(t) \quad \text{are continuous on} \quad 0 \leqq t \leqq T, \tag{1.4}$$

$$v(t)u'(t) - u(t)v'(t) > 0, \qquad 0 \leqq t \leqq T, \tag{1.5}$$

and we always choose the value $\lambda^{1/2}$ such that it has nonnegative real part. This function (1.1) is the density function of a Gaussian Markov process if λ is positive. Examples of the u and v functions are given in [5, p. 304]:

Example 1. Wiener process; $u(t) = t$, $v(t) = 1$, $0 \leqq t \leqq T$.

Example 2. Doob-Kac process (or Brownian bridge); $u(t) = t$, $v(t) = 1 - t$, $0 \leqq t \leqq T < 1$.

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